interacting qubits quantum state transfer n-iQST

OUTLINE:

- One-qubit quantum state transfer
- Quantum dynamical map & random matrix theory approach to n-iQST
- Many-body dynamics in quadratic fermionic models
- Applications: n-QST, quantum batteries, entanglement generation

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Quantum Hiking 2019





 $u^{\scriptscriptstyle b}$ Average fidelity between Kurse, July 18, 2019 a purve state and its image Kurse, July 18, 2019 Quentum Let $\underline{\Psi}(3) = 3 = \underline{\Xi}A; 3A;$ be a quentum openation, i=1 (trace preservin fidelity F(14), (4)) = K+14>1 ~ completely 14) Edla F(14), 8) = 241814) = Proba- $F(1+)(+1) \overline{\Phi}(1+)(+1) = (1+)(+1)(+) = \underbrace{\left[\sum_{i=1}^{\infty} A_i^{\dagger} A_i^{\dagger} = A_i^{\dagger} = A_i^{\dagger} A_i^{\dagger} = A_i^$ $= \sum_{i=1}^{M} \langle +1A_{i}, 1+ \rangle \langle +1A_{i}, 1+ \rangle = \sum_{i=1}^{M} |\langle +1A_{i}, 1+ \rangle|^{2}$ The average (F(1+)<+1, E(1+)(+)) can be rewriten in 4 E Hos term of the Hear measure on U(N) $|4\rangle = 4102, \quad u \text{ ren dom unitory in } (10) \\ \oplus \psi \in Hors : \\ H = (14)(41) \\ = \langle \xi | 1(0) | U A_2 (10) |^2 \rangle_{4 \in U(N)} =$ M < 1(A'2) 00 | The) where A := U A:U is averaged over theunitary group. U(N) C | U2013= IN, and | U1: 12/U2:12 =? ete. see Mello, J. Phys A. 19883 www.unibe.ch





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Polarization-entanglement based QKD over a 96 km-long submarine optical fiber



Distributed quantum computing/QKD









1D Spin-1/2 XX Hamiltonian - JW mapping – Quadratic, spinless fermion Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \frac{J_i}{2} \left(\hat{c}_{i+1}^{\dagger} \hat{c}_i + \hat{c}_i^{\dagger} c_{i+1} \right) + h_i \hat{c}_i^{\dagger} \hat{c}_i$$



The QST average fidelity depends only on the transition amplitude

$$\bar{\mathcal{F}}_{sr}(t) = \frac{1}{2} + \frac{|f_s^r(t)|}{3} + \frac{|f_s^r(t)|^2}{6}$$

Bose, Phys. Rev. Lett. 91, 207901 (2003)

$$\begin{array}{l} \text{n=1 QST IN THE XX MODEL} \\ \hline \text{The single-particle transition amplitude from site s to site r} \\ f_s^r(t) = \sum_{k=1}^N \langle r | \ e^{-it\hat{H}} | s \rangle = \sum_{k=1}^N e^{-i\omega_k t} \phi_{rk} \phi_{ks}^* \\ \hline \text{The QST average fidelity depends only on the transition amplitude} \\ \hline \bar{\mathcal{F}}_{sr}(t) = \frac{1}{2} + \frac{|f_s^r(t)|}{3} + \frac{|f_s^r(t)|^2}{6} \\ \hline \text{Bose, Phys. Rev. Lett. 91, 207901 (2003)} \\ \hline \end{array}$$

$$\begin{array}{l} \text{n=1 QST IN THE XX MODEL} \\ \hline \text{The single-particle transition amplitude from site s to site } r \\ f_s^r(t) = \sum\limits_{k=1}^N \langle r | \ e^{-it\hat{H}} \ | s \rangle = \sum\limits_{k=1}^N e^{-i\omega_k t} \phi_{rk} \phi_{ks}^* \\ \hline f_s^r(t) = \sum\limits_{k=1}^N \langle r | \ e^{-it\hat{H}} \ | s \rangle = \sum\limits_{k=1}^N e^{-i\omega_k t} \phi_{rk} \phi_{ks}^* \\ \hline \text{The QST average fidelity depends only on the transition amplitude} \\ \hline \bar{\mathcal{F}}_{sr}(t) = \frac{1}{2} + \frac{|f_s^r(t)|}{3} + \frac{|f_s^r(t)|^2}{6} \\ \hline \mathcal{F}_{max} = \frac{2}{d+1} \\ \hline \text{For a qubit (d=2)} \\ \text{the maximum fidelity} \\ \text{achievable by LOCC is} \\ \hline \mathcal{P}erturbative \ couplings \\ \hline \text{Vojcik et al. Phys. Rev. A 174, 062316 (2006)} \\ \hline \text{Lorenzo et al., Phys. Rev. A 37, 042313 (2013)} \\ \hline \bar{\mathcal{F}}_{sr}(t^*) = 1 - O\left(Nj^2\right) \\ \hline \left\{ \begin{array}{c} t_e^* = j^{-2} \ \text{even } N \\ t_o^* = \frac{\sqrt{N}}{j} \ \text{odd } N \end{array} \right. \end{array} \right.$$



Perturbatively Perfect (PP) transfer

Excitation transfer occur via <u>Rabi-like oscillations</u> between the sender and the receiver



A single wire for multiple QIP tasks

Motivations:

- The technological challenge of faithful quantum wire;
- The request of scalability of a quantum computer;
- The short coherence times of the coherent dynamics;
- The protection against environmental intrusions;
- The economical costs of a single quantum wire;
- •

A single channel for multiple QIP tasks

Motivations:

- The technological challenge of faithful quantum wire;
- The request of scalability of a quantum computer;
- The short coherence times of the coherent dynamics;
- The protection against environmental intrusions;
- The economical costs of a single quantum wire;
- •

Can perturbative couplings be helpful in this regard?

Task: many-body quantum state transfer



Motivations: the output of a QIP protocol is a n-qubit state transfer of multipartite entanglement many-body properties transfer

Alternative Protocols: parallel/sequential use of a 1-QST use of entangled states as QC PQST QC

$$\langle F \rangle = \frac{1}{\Omega} \int_{\Omega} d\Omega \ F\left(|\Psi\rangle, \rho(t) \right)$$

The problem is to perform the average over all input states (efficient parametrization of an n-qubit state)

Quantum dynamical map & random matrix theory approach

$$\rho^{R} = \Lambda[\rho^{S}] \quad - > \quad \rho^{R} = \sum_{n,m=0}^{d-1} \sum_{i,j=0}^{d-1} A_{ij}^{nm} a_{n} a_{m}^{*} |i| > < j|$$

$$F\left(\left|\Psi\right\rangle,\rho\right) = \left\langle\Psi\right|\rho\left|\Psi\right\rangle = \sum_{ijnm=0}^{d-1} a_{i}^{*}a_{j}a_{n}a_{m}^{*}A_{ij}^{nm},$$

Averages from Random Matrix Theory

$$\left\langle |a_i|^2 \right\rangle = \frac{1}{d} , \quad \left\langle |a_i|^4 \right\rangle = \frac{2}{d(d+1)} , \quad \left\langle |a_i|^2 |a_j|^2 \right\rangle_{i \neq j} = \frac{1}{d(d+1)}$$

$$\frac{Average Fidelity}{Average Fidelity}$$

$$\left\langle F \right\rangle = \frac{1}{d(d+1)} \left(2\sum_{i=0}^{d-1} A_{ii}^{ii} + \sum_{i\neq j=0}^{d-1} A_{ii}^{jj} + 2\Re \left\{ \sum_{i>j=0}^{d-1} A_{ij}^{ij} \right\} \right) .$$

The trivial case $\Phi(t) = \mathbf{I}$, i.e., $A_{ij}^{nm} = \delta_{in}\delta_{jm}$, yielding $\langle F \rangle = 1$.

The LOCC-limit is obtained from the first two contributions to $\langle F \rangle$, which come from those elements of the map A connecting all of the diagonal elements of the density matrices of S and R. The LOCC limit is obtained by setting $A_{ij}^{nm} = \delta_{ij}\delta_{nm}$, yielding $\langle F \rangle = \frac{2}{d+1}$.

The third term, instead, is due to off-diagonal map elements, connecting input to output coherences. It has, thus, a purely quantum origin, and it disappears for a classical map. Its contribution scales as $\frac{d-1}{d+1}$.

Variance of the Fidelity

$$\left(\Delta F\right)^2 = \left\langle F^2 \right\rangle - \left\langle F \right\rangle^2$$

$$F^{2}(|\Psi\rangle,\rho) = \sum_{ijnmpqrs=0}^{d-1} a_{i}^{*}a_{j}a_{n}a_{m}^{*}a_{p}^{*}a_{q}a_{r}a_{s}^{*}A_{ij}^{nm}A_{pq}^{rs}.$$

$$\begin{split} \left< F^2 \right> &= \frac{1}{d(d(+1)(d+2)(d+3)} \times \\ &\sum_{i,m,p,s=0}^{d-1} \left((A_{ii}^{mm} + A_{im}^{im})(A_{pp}^{ss} + A_{ps}^{ps}) + (A_{ii}^{pm} + A_{ip}^{im})(A_{pm}^{ss} + A_{ps}^{ms}) + \right. \\ &\left. (A_{ii}^{sm} + A_{is}^{im})(A_{pm}^{ps} + A_{pp}^{ms}) + (A_{im}^{pm} + A_{ip}^{mm})(A_{pi}^{ss} + A_{ps}^{is}) + \right. \\ &\left. (A_{im}^{sm} + A_{is}^{mm})(A_{pi}^{ps} + A_{pp}^{is}) + (A_{ip}^{sm} + A_{is}^{pm})(A_{pi}^{ms} + A_{pm}^{is}) \right) \,. \end{split}$$

 $\rho^R = \Lambda[\rho^S]$

The explicit expression for the dynamical map of a U(1)-symmetric Hamiltonian (where the quantum channel and the receiver is initialized in the fully polarized state)

$ \begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{02} \\ \rho_{03} \\ \rho_{10} \end{pmatrix} $		$\begin{pmatrix} A_{00}^{00} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$0\\ A_{01}^{01}\\ A_{02}^{01}\\ 0\\ 0\\ 0\\ 0$	$egin{array}{c} 0 \ A^{02}_{01} \ A^{02}_{02} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ $	$egin{array}{c} 0 \\ 0 \\ 0 \\ A^{03}_{03} \\ 0 \\ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ A^{10}_{10} \\ 0 \end{array}$	$A^{11}_{00} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 11$	$A^{12}_{00} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1^{12}$	$egin{array}{c} 0 \ A^{13}_{01} \ A^{13}_{02} \ 0 \ 0 \ 0 \ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ A^{20}_{10} \\ 0 \end{array}$	$A^{21}_{00} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4^{21}$	A_{00}^{22} 0 0 0 0 0	$egin{array}{c} 0 \ A^{23}_{01} \ A^{23}_{02} \ 0 \ 0 \ 0 \ 0 \end{array}$	0 0 0 0 0	$egin{array}{c} 0 \\ 0 \\ 0 \\ A^{31}_{10} \\ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ A^{32}_{10} \\ 0 \end{array}$	$A_{00}^{33} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1^{33}$	$ \begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{02} \\ \rho_{03} \\ \rho_{10} \end{pmatrix} $
$ ho_{11} ho_{12} ho_{13} ho_{20} ho_{21} ho_{22}$	_	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	$egin{array}{c} 0 \\ 0 \\ A_{20}^{10} \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} A_{11}^{11} \\ A_{12}^{11} \\ 0 \\ 0 \\ A_{21}^{11} \\ A_{22}^{11} \\ A_{22}^{12} \end{array}$	$\begin{array}{c} A_{11}^{12} \\ A_{12}^{12} \\ 0 \\ 0 \\ A_{21}^{12} \\ A_{22}^{12} \\ A_{22}^{12} \end{array}$	$egin{array}{c} 0 \\ 0 \\ A^{13}_{13} \\ 0 \\ 0 \\ 0 \\ 0 \\ 4^{13} \end{array}$	$egin{array}{c} 0 \\ 0 \\ A^{20}_{20} \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} A_{11}^{21} \\ A_{12}^{21} \\ 0 \\ 0 \\ A_{21}^{21} \\ A_{22}^{21} \\ \end{array}$	$\begin{array}{c} A_{11}^{22} \\ A_{12}^{22} \\ 0 \\ 0 \\ A_{21}^{22} \\ A_{22}^{22} \\ 2 \\ 2 \end{array}$	$egin{array}{c} 0 \\ 0 \\ A^{23}_{13} \\ 0 \\ 0 \\ 0 \\ 0 \\ A^{23} \end{array}$	0 0 0 0 0 0	$egin{array}{c} 0 \\ 0 \\ A^{31}_{20} \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ A_{20}^{32} \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} A_{11}^{33} \\ A_{12}^{33} \\ 0 \\ 0 \\ A_{21}^{33} \\ A_{22}^{33} \\ A_{22}^{33} \\ 2 \\ \end{array}$	$ ho_{11} ho_{12} ho_{13} ho_{20} ho_{21} ho_{22}$
$ \begin{array}{c} \rho_{23} \\ \rho_{30} \\ \rho_{31} \\ \rho_{32} \\ \rho_{33} \end{array} \right _{N^{-1}} \\ N^{-1} \\$	-1,N	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	$A_{23}^{13} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	$egin{array}{c} A_{23}^{23} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$egin{array}{c} 0 \ A^{30}_{30} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \$	$\begin{array}{c} 0 \\ 0 \\ A^{31}_{31} \\ A^{31}_{32} \\ 0 \end{array}$	$\begin{array}{c} 0 \\ 0 \\ A_{31}^{32} \\ A_{32}^{32} \\ 0 \end{array}$	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ A_{33}^{33} \end{array}$	$ \begin{pmatrix} \rho_{23} \\ \rho_{30} \\ \rho_{31} \\ \rho_{32} \\ \rho_{33} \end{pmatrix}_{1,2} $

 $\begin{aligned} A_{00}^{00} &= 1 \ , \ A_{00}^{11} = 1 - \left| f_1^{N-1} \right|^2 - \left| f_1^N \right|^2 \ , \ A_{00}^{22} &= 1 - \left| f_2^{N-1} \right|^2 - \left| f_2^N \right|^2 \ , \\ A_{00}^{33} &= 1 - \left| f_{12}^{mN-1} \right|^2 - \left| f_{12}^{mN} \right|^2 - \left| f_{12}^{N-1N} \right|^2 \ , \\ A_{00}^{12} &= -f_1^{N-1} \left(f_2^{N-1} \right)^* - f_1^N \left(f_2^N \right)^* \ , \ A_{00}^{21} &= -f_2^{N-1} \left(f_1^{N-1} \right)^* - f_2^N \left(f_1^N \right)^* \ , \end{aligned}$

N-QST over independent channels



Product states are transferred better than entangled states





many-body dynamics in terms of one-body dynamics in bilinear models

$$\begin{array}{c} \underbrace{\prod_{i=1}^{12 \dots n_{s}} \underbrace{\lim_{i \neq 1}^{n_{s} \dots 2} 1}_{j}}_{\hat{H} = \sum_{i=1}^{N} \frac{J_{i}}{2}} \left(\hat{c}_{i+1}^{\dagger} \hat{c}_{i} + \hat{c}_{i}^{\dagger} c_{i+1} \right) + h_{i} \hat{c}_{i}^{\dagger} \hat{c}_{i} \rightarrow \sum_{k=1}^{N} \omega_{k} \hat{c}_{k}^{\dagger} \hat{c}_{k} \\ \hat{H} = \sum_{i=1}^{N} \frac{J_{i}}{2} \left(\hat{c}_{i+1}^{\dagger} \hat{c}_{i} + \hat{c}_{i}^{\dagger} c_{i+1} \right) + h_{i} \hat{c}_{i}^{\dagger} \hat{c}_{i} \rightarrow \sum_{k=1}^{N} \omega_{k} \hat{c}_{k}^{\dagger} \hat{c}_{k} \\ \text{One-body transition amplitude} \\ f_{i}^{j}(t) = \sum_{k=1}^{N} e^{-i\omega_{k}t} \phi_{jk} \phi_{ki}^{*} \\ \hat{f}_{i}^{n_{s}}(t) = \sum_{k=1}^{N} e^{-i\omega_{k}t} \phi_{jk} \phi_{ki}^{*} \\ \text{persymmetric matrix} \\ \text{real (imaginary) for i+j even (odd)} \\ \text{symmetric spectrum around the center implies} \\ 1) \frac{n_{s} \left(n_{s} + 1\right)}{2} \quad \text{distinct elements} \\ 2) \text{ one's on the main and skew diagonal} \\ 3) \left\lceil \frac{n_{s}}{2} \rceil \quad \text{distinct terms on the main diagonal} \\ \end{array} \right| \begin{array}{c} \frac{Mirror-symmetric}{Hamiltonian:} \\ \frac{Mirror-symmetric}{Hamilto$$









-



Variance vs average fidelity (n-QST) across independent channels for product states (dotted lines) and arbitrary states (continuous lines)

Variance vs average Fidelity for independent channels (continuous lines) and for an interacting channel (dotted lines)



Length of wires that are equivalent mod(number of senders) have the same behaviour w.r.t. excitation transfer

Number of Excitations	Number of Resonant Modes	Length of the wire								
1	0 1	2n $2n+1$								
2	0 0 2	3n $3n+1$ $3n+2$								
3	0 1 0 3	$4n 4n{+}1 4n{+}2 4n{+}3$								
4	0 0 0 0 4	5n $5n+1$ $5n+2$ $5n+3$ $5n+4$								
5	0 1 2 1 0 5	6n $6n+1$ $6n+2$ $6n+3$ $6n+4$ $6n+5$								





<u>2-particle routing in network configuration:</u>



2-particle routing in network configuration:



- Length of wire n_w=3n+2 (fulfills the energy resonance condition);
- Postion of the switchable couplings n, #2n+1 (fulfills the equal-amplitude condition);
- Perturbatively perfect 2-excitation transfer in O(J₀⁻¹) time for every contact point.

2-particle routing in network configuration:



- n_w/2-even (n_w-1)/2-odd possible receivers
- postion of the permanent couplings depending on J_i (fulfills the equal-amplitude condition);
- Perturbatively perfect 2-excitation transfer in O(J₀⁻¹) time for every contact point.

2-particle routing in network configuration:



- n_w/2-even (n_w-1)/2-odd possible receivers
- postion of the permanent couplings depending on J_i (fulfills the equal-amplitude condition);
- Perturbatively perfect 2-excitation transfer in O(J₀⁻¹) time for every contact point.







Questions:

- Does the protocol work for very long chains? I.e., is there a regime where the dynamics is governed by the single-particle spectrum?
- Is there any analogy between spin dynamics and (interacting) fermion/boson dynamics?

CONCLUSIONS

- n-QST vs. n-iQST
- First and second moment of the fidelity PDF

• Applications to quantum batteries and multi-qubit bipartite entanglement generation

<u>Outlooks:</u>

- PDF of fidelity
- n-iQST for arbitrary n
- Characterisation of multipartite entanglement generation *Quantum transfer of interacting qubits*, T. J. G. Apollaro, S. Lorenzo, F. Plastina, M. Consiglio, Karol Zyczkowski, https://doi.org/10.48550/arXiv.2205.01579 *Quantum map approach to entanglement transfer and generation in spin chains*, S. Lorenzo, F. Plastina, M. Consiglio, T. J. G. Apollaro https://arxiv.org/abs/2112.02348 *Perturbatively-perfect many-body transfer*, Chetcuti, Sanavio, Lorenzo, Apollaro, New J. Phys. 22, 033030 (2020)



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