Multicopy entanglement detection from the $\underbrace{\text{symmetric}}_{\text{(permuting stuff)}}$ group ring ($\mathbb{C} S_n$)

$A \in M_{n \times n}$

& F. Huber

1) $\mathrm{Det}(A) = \sum_{\sigma \in S_n} \mathrm{sign}(\sigma) \prod_i A_{i\sigma(i)}$

$\mathrm{Perm}(A) = \sum_{\sigma \in S_n} (+1) \prod_i A_{i\sigma(i)}$

$$\boxed{\mathrm{Imm}_\lambda(A) = \sum_{\sigma \in S_n} \chi_\lambda(\sigma) \prod_i A_{i\sigma(i)}}$$

$A \geq 0$

$\chi_\lambda(e)\, \mathrm{Imm}_\lambda(A) \geq \mathrm{Imm}_\mu(A)\, \chi_\mu(e)$

$$\boxed{\mathrm{Perm}(A) \geq \mathrm{Det}(A)}$$

$$0 \leq \mathrm{Perm}(A) - \mathrm{Det}(A) = \sum_\sigma \prod_i A_{i\sigma(i)} - \sum_\sigma \mathrm{sign}(\sigma) A_{i\sigma(i)} =$$

$$A = \left(\langle v_i | v_j \rangle\right)$$

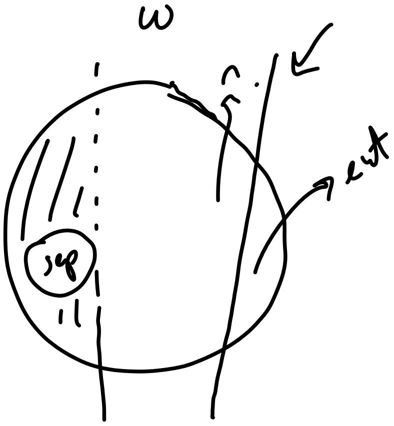
$$= \sum_\sigma \prod_i \underbrace{\langle v_i | \sigma^{-1} | v_i \rangle}_{\mathrm{Tr}[\sigma^{-1} |v_i\rangle\langle v_i|]} - \sum_\sigma \mathrm{sign}(\sigma) \prod_i \langle v_i | \sigma^{-1} | v_i \rangle =$$

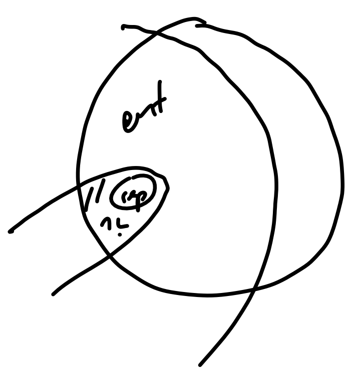
$$= \mathrm{Tr}\left[\sum_\sigma \sigma^{-1} \bigotimes_i |v_i\rangle\langle v_i| - \sum_\sigma \mathrm{sign}(\sigma)\, \sigma^{-1} \bigotimes_i |v_i\rangle\langle v_i|\right] = \mathrm{Tr}\left[(P_+ - P_-)\, n! \underbrace{\bigotimes_{i=1}^{n} |v_i\rangle\langle v_i|}_{\text{lin. comb}}\right]$$

$$0 \leq \mathrm{Tr}\left[\underbrace{(P_+ - P_-)}_{W} \rho_{sep}\right]$$

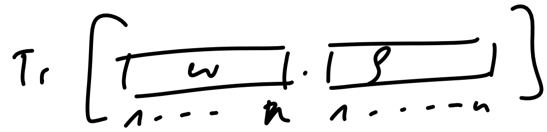
[Maassen, Kuemmerer 2007]

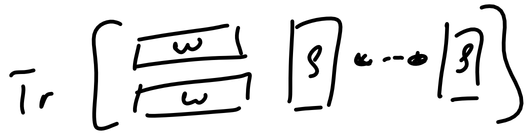
$\rho = \sum_a p_a \bigotimes_i |v_i^a\rangle\langle v_i^a|$

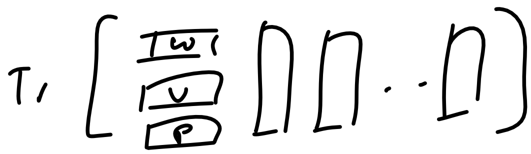




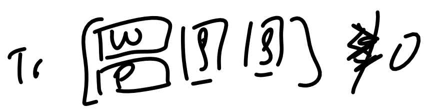
$I_{m\lambda}(A) - I_{m\mu}(A) \geq 0 \Rightarrow \frac{p_\lambda}{\chi_\lambda(A)^2} - \frac{p_\mu}{\chi_\mu^2} = \omega.$

$\mathrm{Tr}[\omega \cdot \rho] \rightarrow$ 

$\mathrm{Tr}[(\omega_A \otimes \omega_B)_{1\dots n} \cdot \rho_{AB,1} \otimes \dots \otimes \rho_{AB,n}) =$ 

$\rightarrow$ NO detect unknown 

$\rightarrow \exists$ ex: $\mathrm{Tr}[\omega\rho] \geq 0$



Props:

1. Inv. local unitaries (Shur-Weil)
2. ω depends only spectr(ρ_{AB}), spec(ρ_A), spec(ρ_B)