

$$U(g) = e^{-igJ} \quad \searrow \quad J \text{ abs.}$$

P

$$\underline{U(g) H U^\dagger(g) = H}$$

$$[H, J] \stackrel{?}{=} 0 \quad \langle j \rangle = 0$$

$$\phi \quad U(\rho) = e^{-i\rho J}$$

$$\rho_S(t) = \phi(t) [\rho_S(0)]$$

$$\mu(g) [\rho_S] = U(g) \rho_S U^\dagger(g)$$

$$\mu(\rho) \phi \mu^\dagger(\rho) = \phi$$

$$[\phi, \mu(\rho)]$$

$$[U(p), \phi] = 0 \quad \Downarrow$$

$$\begin{aligned} & \phi[U(p) P_5 U^\dagger(p)] \\ &= U(p) \phi[P_5] U^\dagger(p) \end{aligned}$$

$$\phi(t) = e^{\mathcal{L}t}$$

$$\begin{aligned} \mathcal{L}_P &= -i[\omega a^\dagger a, P_5] \quad \downarrow \\ &+ \gamma (a P_5 a^\dagger - \frac{1}{2} \{a^\dagger a, P_5\}) \end{aligned}$$

$$U(p) = e^{i p a^\dagger a} \quad \left. \begin{array}{l} \mu(p) \\ \downarrow a^\dagger a \end{array} \right\}$$

$$[\mu(p), \mathcal{L}] = 0 \quad \downarrow a^\dagger a$$

$$[H_{\text{ext}}, \mathcal{J}] = 0$$

$$[L_k, \mathcal{J}] = 0 \quad \forall k$$

$$[\Phi, \mu_f] = 0$$

$$\nearrow \mathcal{H}_S \otimes \mathcal{H}_E \quad U_I$$

$$\Phi[\rho_S] = \text{Tr}_E \left[ U_I \rho_S \otimes |Y_E\rangle\langle Y_E| U_I^\dagger \right]$$

$$[\mathcal{J}_G, U_I] = 0?$$

$$\begin{aligned}
 & \text{N.B. } \tilde{U}(\rho) \neq \tilde{U}(\rho, t) \\
 & U(\rho) \otimes \tilde{U}(\rho) \stackrel{-iH_{\pm}t}{=} \underbrace{e^{-iH_{\pm}t}}_{=} \underbrace{U(\rho)} \otimes \underbrace{|\psi_s\rangle \langle \psi_s|}_{\text{forall } \psi_s}
 \end{aligned}$$

$$A |v\rangle$$

$$K_2(A, |v\rangle) = \text{Span of}$$

$$|v\rangle, A|v\rangle, A^2|v\rangle, \dots, A^n|v\rangle$$

$$\exists \alpha_0 \uparrow A^{\alpha_0} |v\rangle$$

$$A^\dagger = A$$
$$A^\dagger = A^{-1}$$

$$\Phi(t) |P_S\rangle = \text{Tr}_E [U_I(t)$$

$$P_S \otimes |\psi\rangle \langle \psi| U_I^\dagger(t)]$$

$$\downarrow$$
$$U_I(t) = e^{-iH_I t}$$

$$[\Phi(t), U(\rho)] = 0 \quad \forall t$$

$$U(\rho) \rightarrow e^{-i\rho J}$$

$$U_g^\dagger H_I U_g |k_1\rangle = H_I |k_1\rangle$$

$$\left\{ \begin{array}{l} U_{\mathcal{I}} H_{\mathcal{I}} U_{\mathcal{I}}^{\dagger} \\ K_{\mathcal{I}} = \sum_{j=1}^m \kappa_{\alpha_j} (H_{\mathcal{I}}, |Y_{S_j}\rangle \langle Y_{S_j}| \oplus |Y_E\rangle \langle Y_E|) \end{array} \right.$$

basis  
↑

$$H_{\mathcal{I}} = H_{\mathcal{I} //} \oplus H_{\mathcal{I} \perp}$$

$$U(\mathcal{I}) = U(\mathcal{I} //) \oplus U(\mathcal{I} \perp)$$