$U(g) = e^{-igJ}$ $\rightarrow$ J obs.

P

$\underline{U(g) H U^{\dagger}(g) = H}$

$\Updownarrow$

$[H, J] = 0$ $\quad \langle \dot{J} \rangle = 0$

ϕ $\qquad U(g) = e^{-igJ}$

$\rho_S(t) = \phi(t)[\rho_S(0)]$

$\mathcal{U}(g)[\rho_S] = U(g)\rho_S U^{\dagger}(g)$

$\mathcal{U}(g)\phi\,\mathcal{U}^{\dagger}(g) = \phi$

$[\phi, \mathcal{U}(g)]$

$$[\mathcal{U}(\varphi), \Phi] = 0$$

$$\Phi[U(\varphi)\rho_s U^\dagger(\varphi)]$$

$$= U(\varphi)\Phi[\rho_s]U^\dagger(\varphi)$$

$$\Phi(t) = e^{\mathcal{L}t}$$

$$\mathcal{L}\rho_s = -i[\omega a^\dagger a, \rho_s]$$

$$+ \gamma\left(a\rho_s a^\dagger - \frac{1}{2}\{a^\dagger a, \rho_s\}\right)$$

$$U(\varphi) = e^{i\varphi a^\dagger a} \qquad \mathcal{U}(\varphi)$$

$$a^\dagger a$$

$$[\mathcal{U}(\varphi), \mathcal{L}] = 0$$

$$[H_{\text{eff}}, J] = 0$$

$$[L_k, J] = 0 \quad \forall k$$

$$[\Phi, \mathcal{U}_g] = 0$$

$$\mathcal{H}_S \otimes \mathcal{H}_E \qquad U_I$$

$$\Phi[\rho_S] = \mathrm{Tr}_E\left[U_I\, \rho_S \otimes |\psi_E\rangle\langle\psi_E|\, U_I^\dagger\right]$$

$$[J_G, U_I] = 0 \; ?$$

$\mathcal{U}(g)$ $\quad \tilde{U}(g) \neq \tilde{U}(g,t)$

$$U(g) \otimes \tilde{U}(g)\, e^{-iH_I t} |\psi_S\rangle \otimes |\psi_E\rangle$$

$$= e^{-iH_I t}\, U(g) |\psi_S\rangle \otimes |\psi_E\rangle$$

$\forall t \quad \forall g \quad \forall |\psi_S\rangle$

$A \quad |v\rangle$

$K_r(A, |v\rangle) = \text{Span}\{$

$|v\rangle, A|v\rangle, A^2|v\rangle, \ldots, A^r|v\rangle\}$

$\exists\, r_0 \quad A^{r_0}|v\rangle$

$A^\dagger = A$

$A^\dagger = A^{-1}$

$\phi(t)[\rho_S] = Tr_E[U_I(t)$

$\rho_S \otimes |\psi_E\rangle\langle\psi_E| U_I^\dagger(t)]$

$U_I(t) = e^{-iH_I t}$

$[\phi(t), U(g)] = 0 \quad \forall t$

$U(g)$

e^{-igJ}

$U_g^\dagger H_I U_g \big|_{K_1} = H_I \big|_{K_1}$

$U_g H_I U_g$

basis

$$K_{\parallel} = \sum_{J=1}^{n} K_{\text{raj}}\left(H_I, |\psi_{S_J}\rangle \otimes |\psi_E\rangle\right)$$

$$H_I = H_{I\parallel} \oplus H_{I\perp}$$

$$U(g) = U(g)_{\parallel} \oplus U(g)_{\perp}$$