When the Born rule is not enough

Part I: Foreseeing future outcomes of measurements on one system

&

Part II: Repeated measurements on Unruh-DeWitt detectors in (3+1) Minkowski spacetime

Quantum Hiking 2022

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Summary

Part I

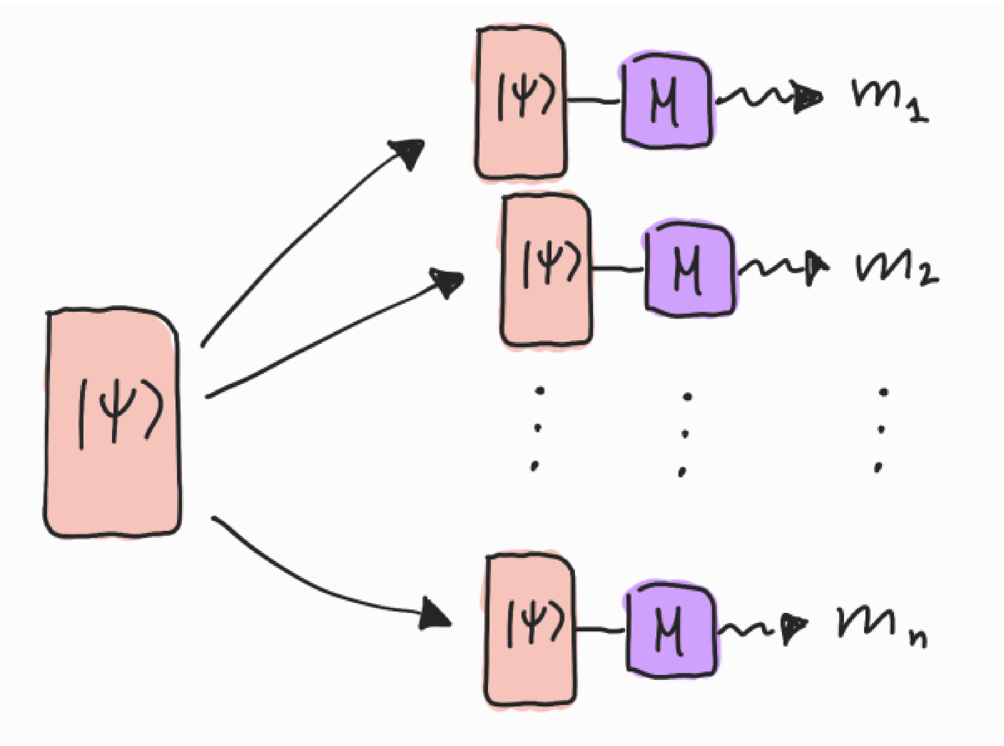
- Born rule
- Repeated Measurements (RM)
- Almost-Born probabilities

Part II

- Unruh-DeWitt detectors (UDW)
- RM on UDW
- Almost-Born rule for UDW

The Born rule

$(\hat{E}_i, m_i)$ $|\psi\rangle$



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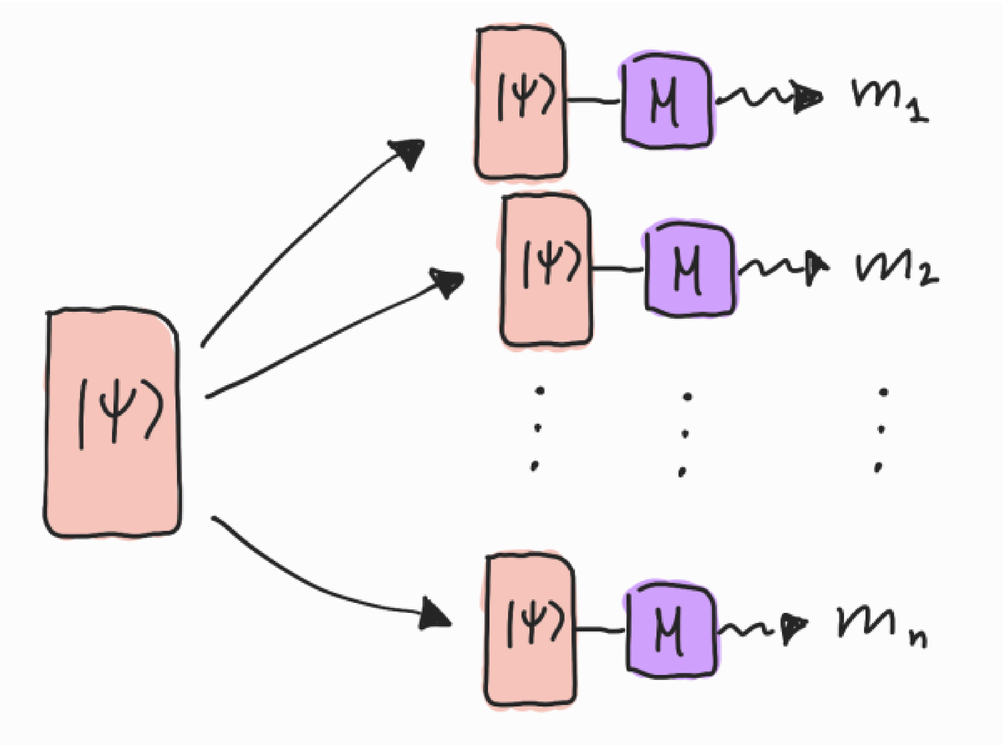
$$\frac{N(m_k)}{\sum_i N(m_i)} \longrightarrow p_{m_k} = \langle\psi|\, \hat{E}_k \,|\psi\rangle$$



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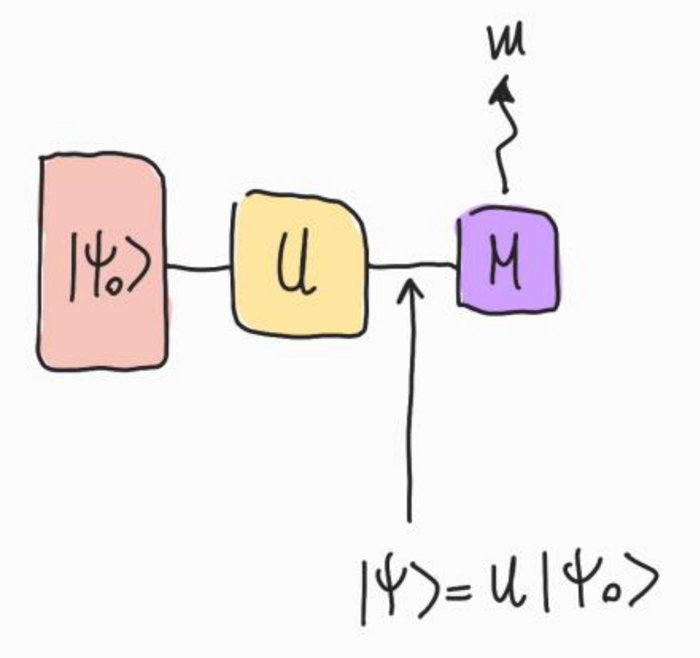

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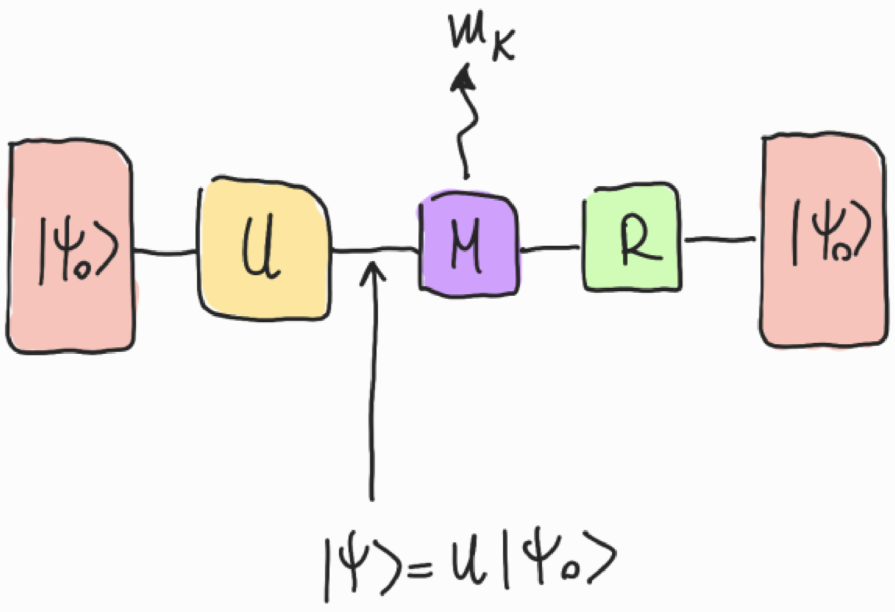

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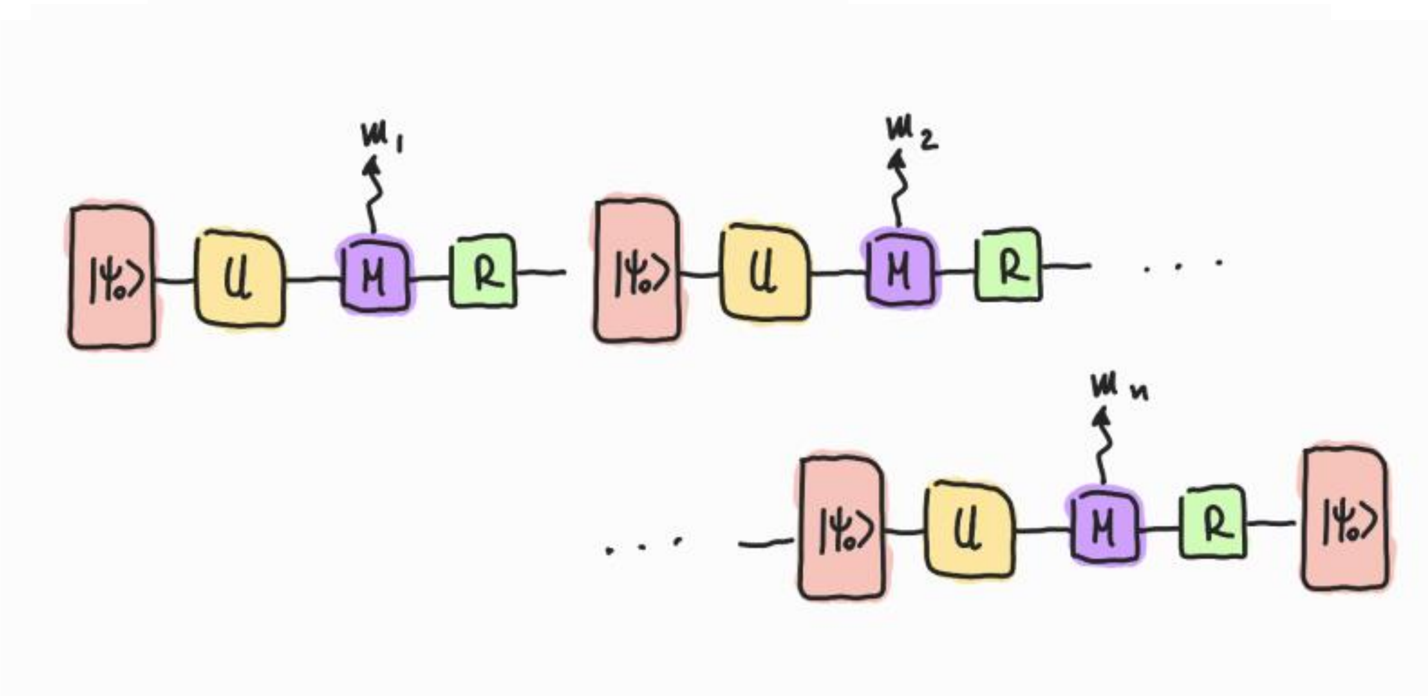
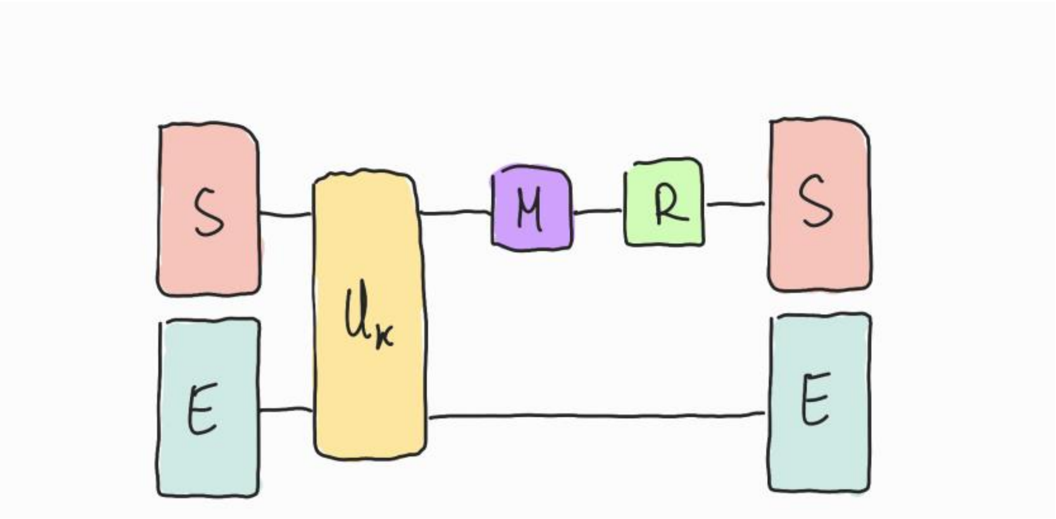


RM scheme

$$\{\mathcal{U}_1, \ldots, \mathcal{U}_n\}$$

$$|g\rangle \otimes |f\rangle \to |g\rangle \otimes \left(\hat{V}_m(k)[f]\,|f\rangle\right)$$

$$\hat{V}_{m_k}(k)[f] = \frac{\langle m_k|\,\hat{\mathcal{U}}_k\,|g\rangle}{\sqrt{p_{m_k}(k)[f]}}$$



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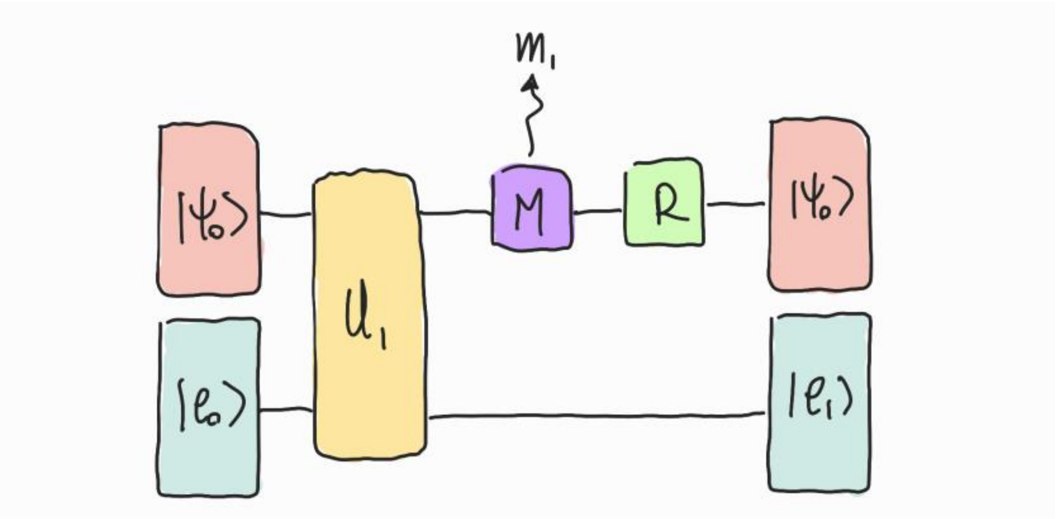

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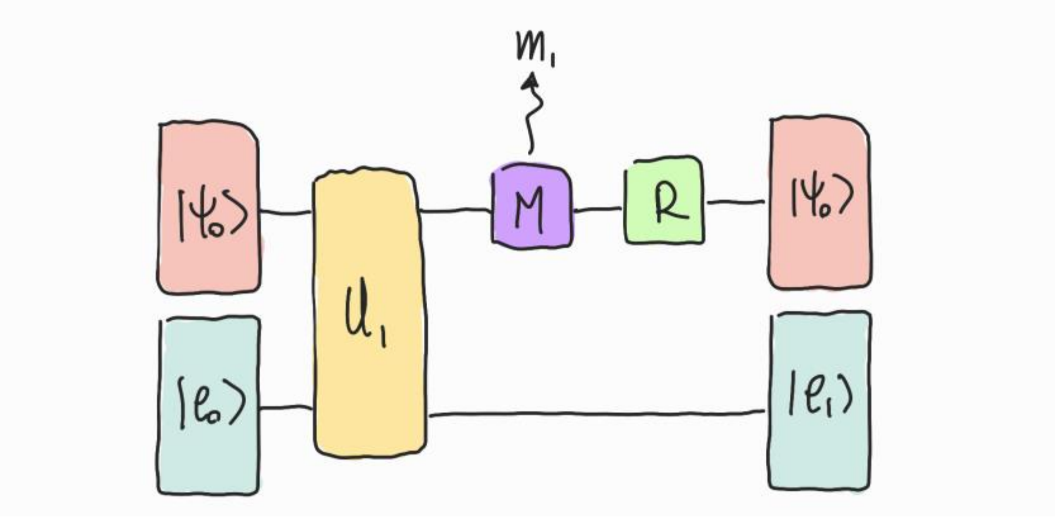

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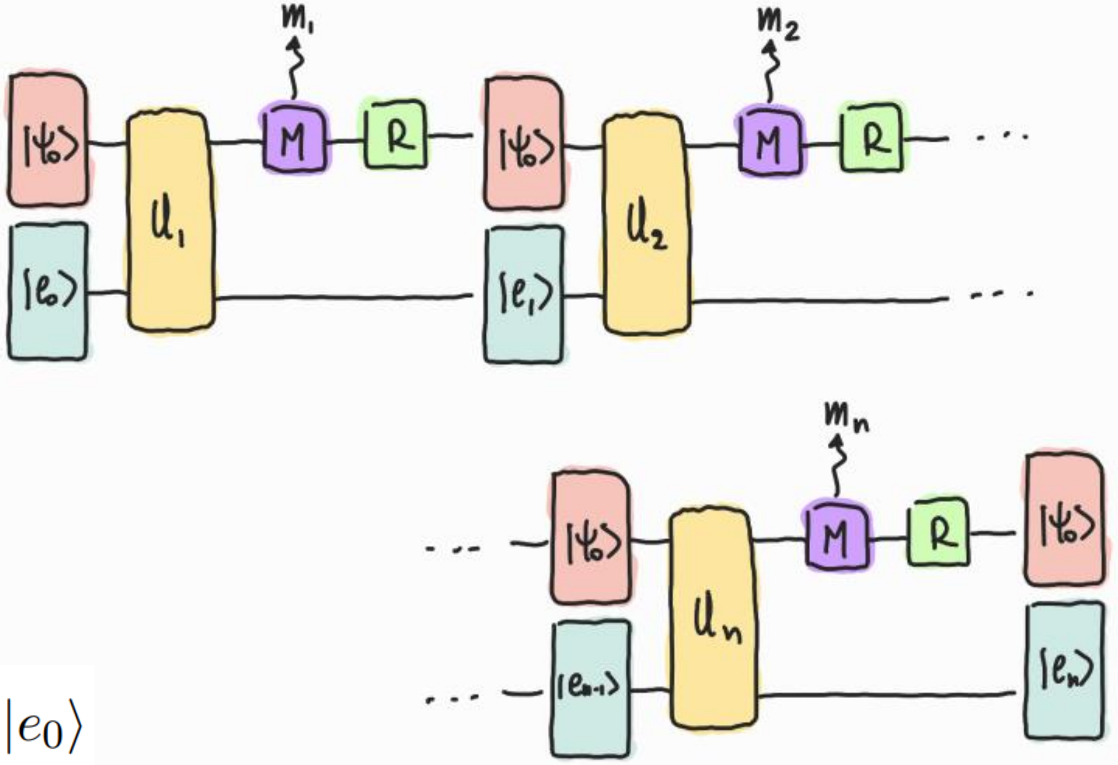

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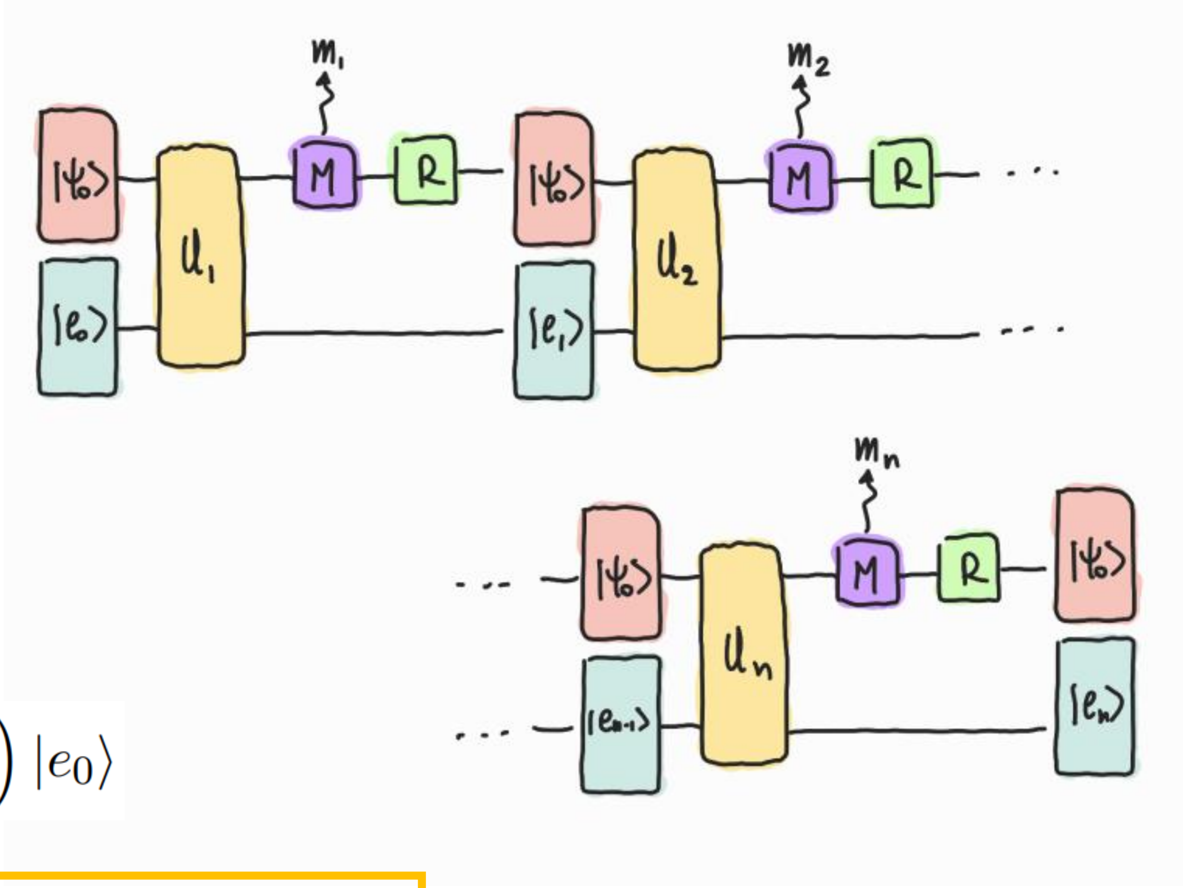
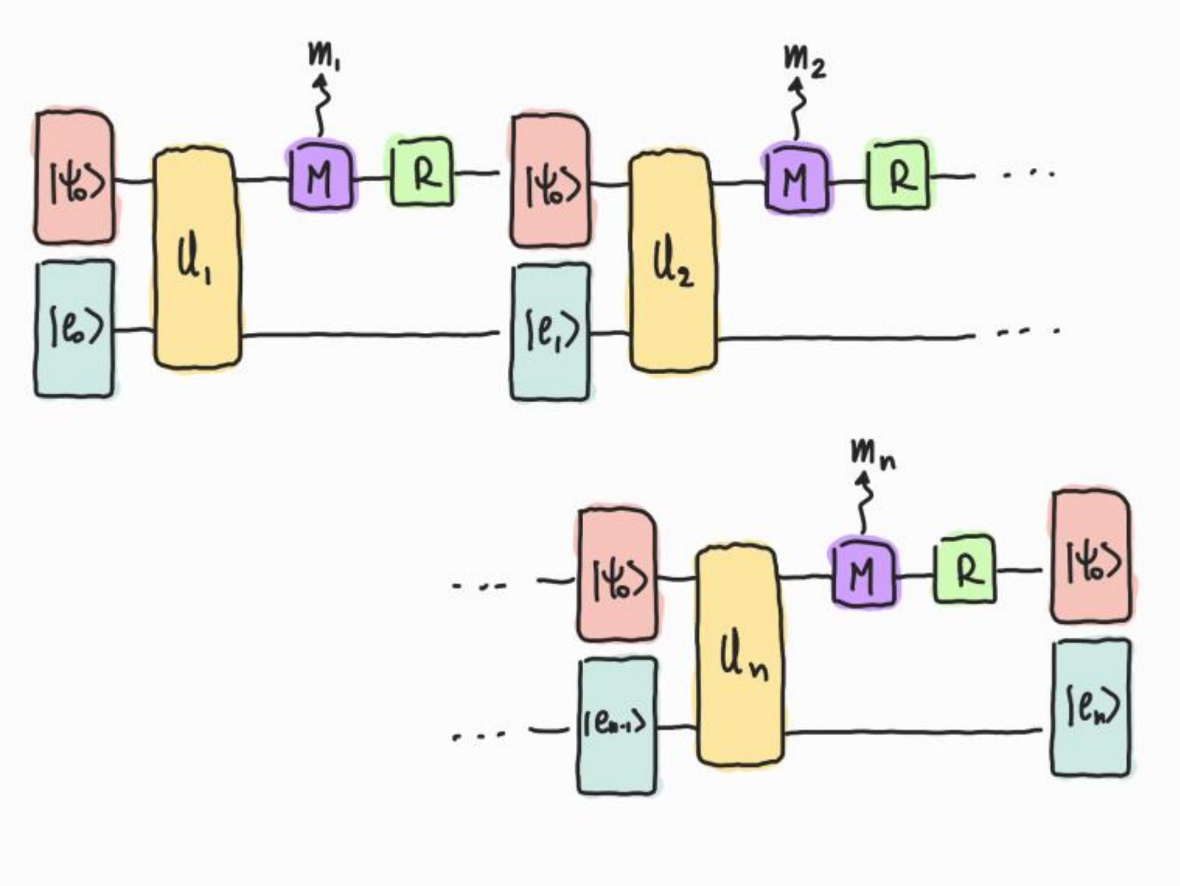


$$\frac{N(m_k)}{\sum_i N(m_i)} \longrightarrow p_{m_k} = \langle\psi|\,\hat{E}_k\,|\psi\rangle$$

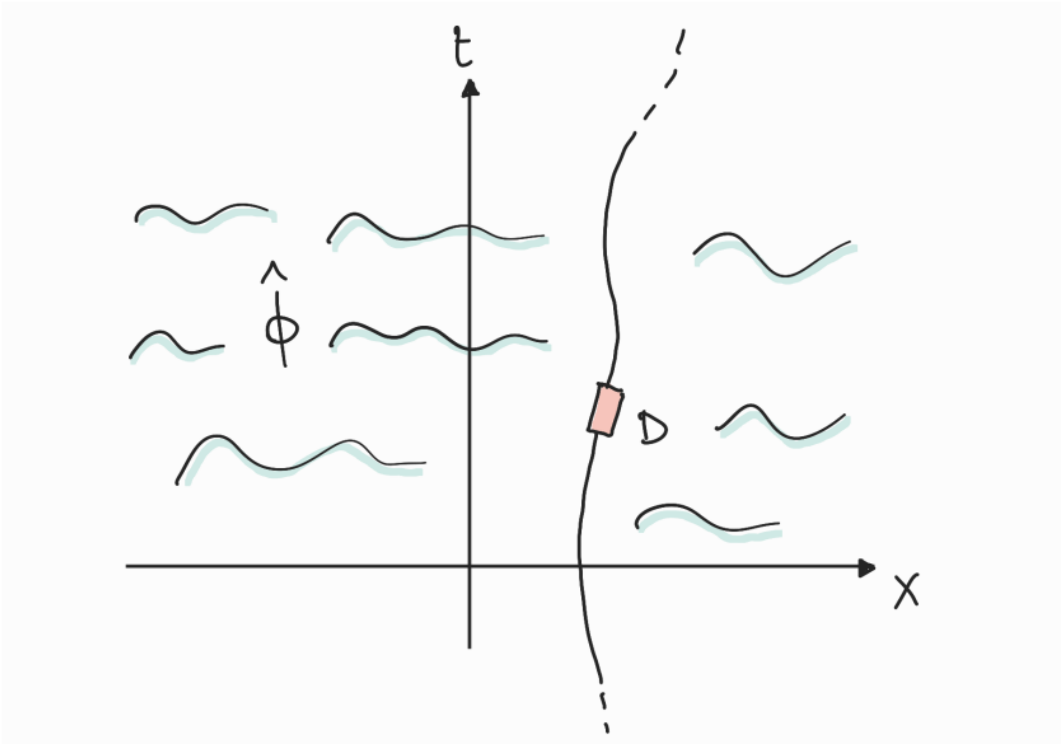
Almost-Born probabilities

$$\hat{\mathcal{U}}_k = \hat{U} \otimes \mathbb{I} + \epsilon \sum_l \hat{A}_l \otimes \hat{B}_l(k) + \epsilon^2 \sum_l \hat{C}_l \otimes \hat{D}_l(k) + O(\epsilon^3)$$



$$p_m(k)[f] = p_m + \epsilon Q_m^{(1)}(k)[f] + \epsilon^2 Q_m^{(2)}(k)[f] + O(\epsilon^3)$$

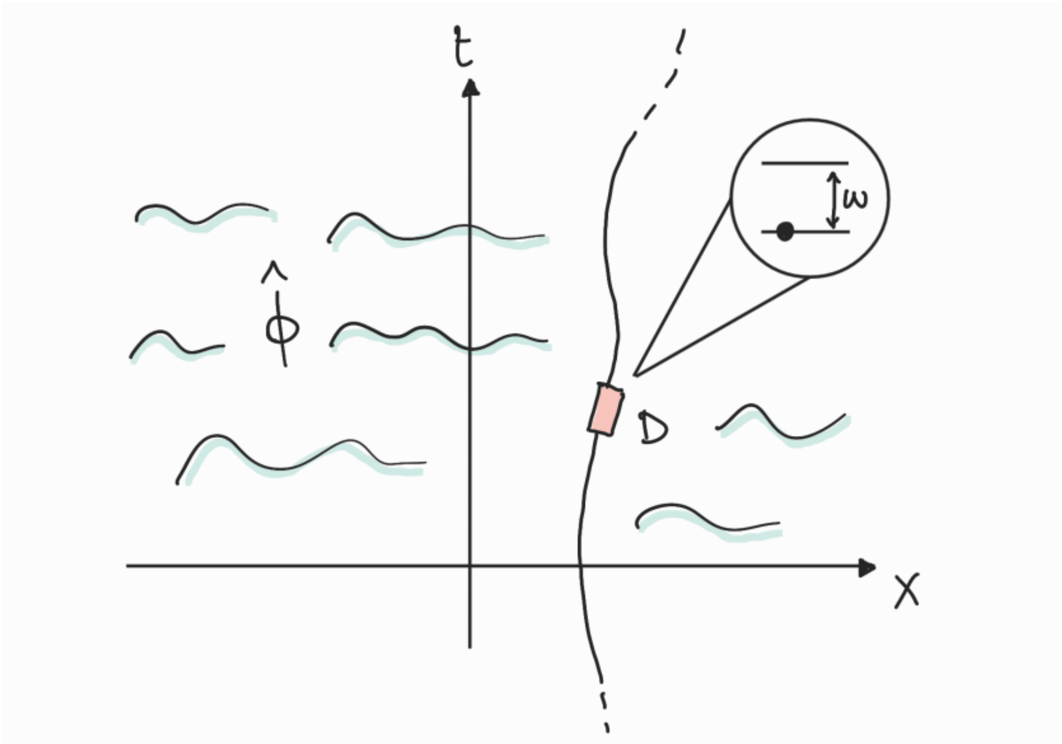
Unruh-DeWitt detectors



$$S = D \cup \phi \;\Rightarrow\; \mathcal{H} = \mathcal{H}_D \otimes \mathcal{H}_\phi$$

Unruh-DeWitt detectors

$$X(\tau) = (t(\tau), \mathbf{x}(\tau)) \ , \ \hat{H}_D = \omega \left| e \right\rangle \left\langle e \right|$$



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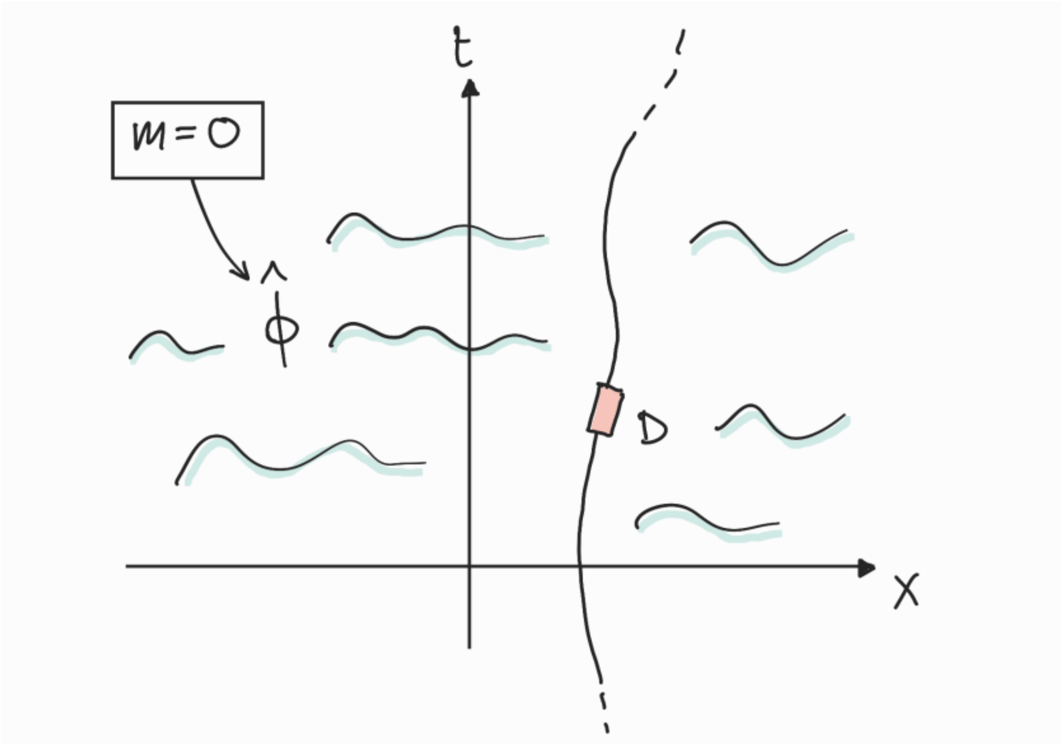
$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \ \Rightarrow \ \hat{H}_\phi = \int \left(\frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\nabla \hat{\phi})^2 \right)$$



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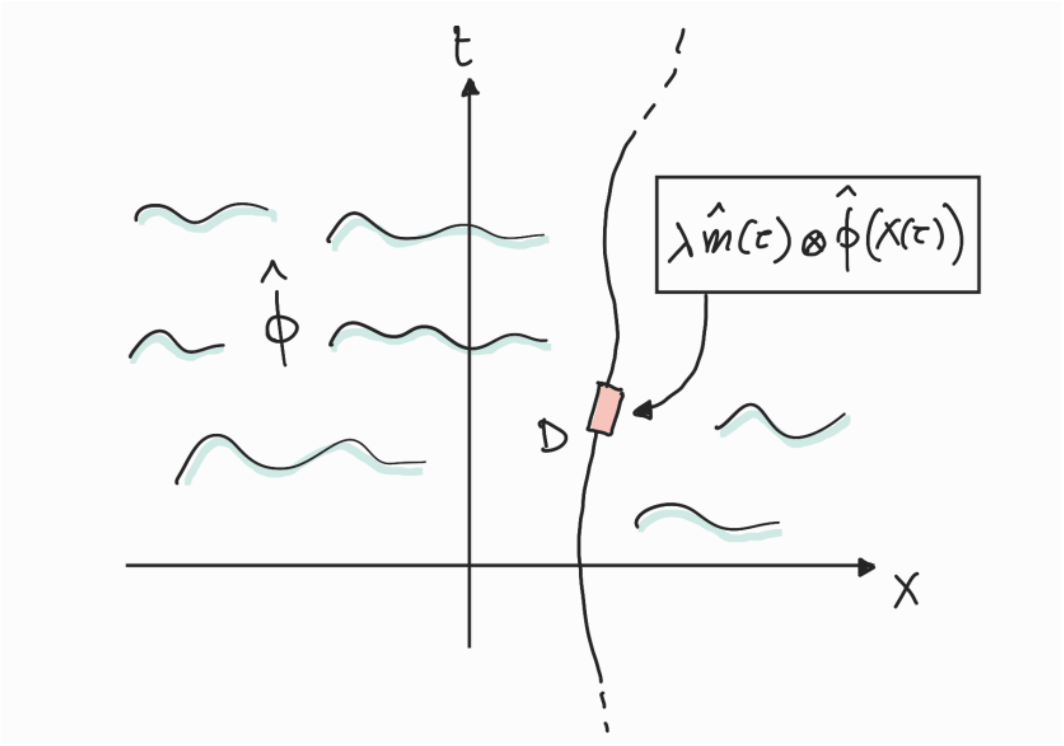
$$\hat{H}_{int}(\tau) = \lambda \hat{m}(\tau) \otimes \hat{\phi}(X(\tau))$$



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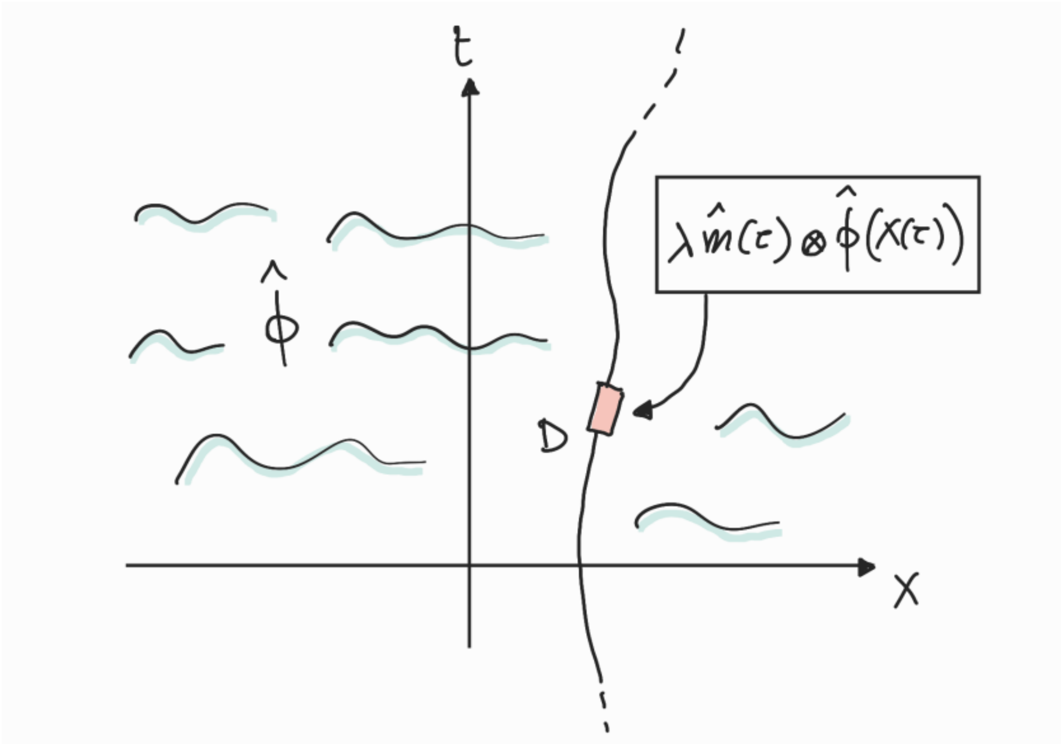

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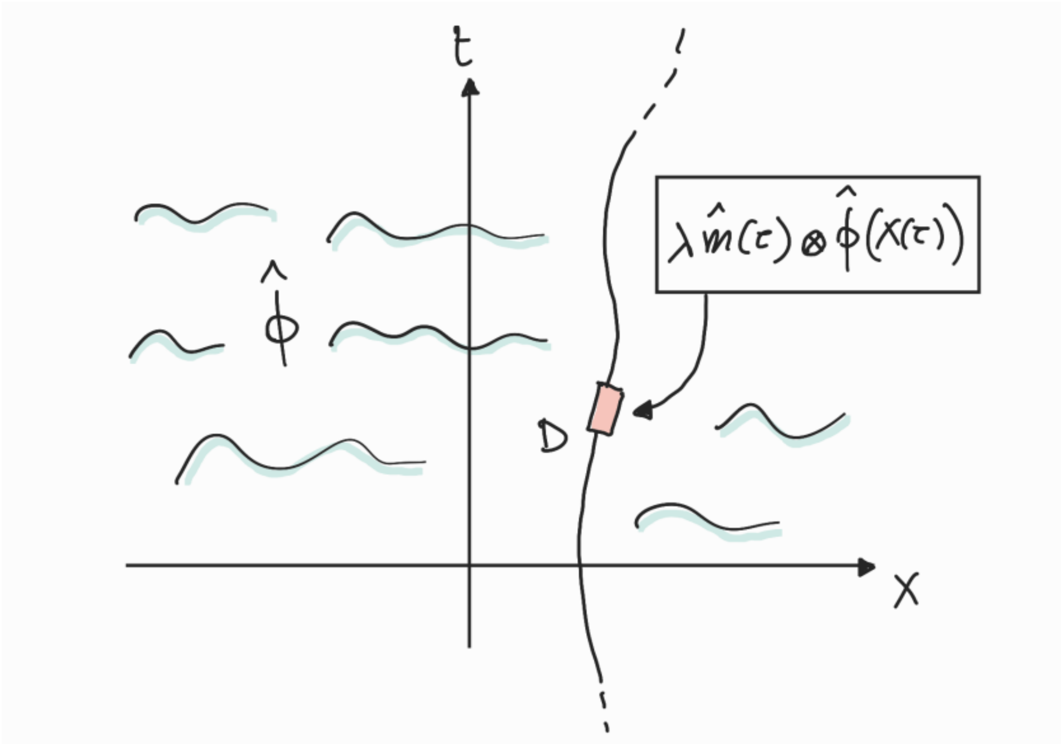

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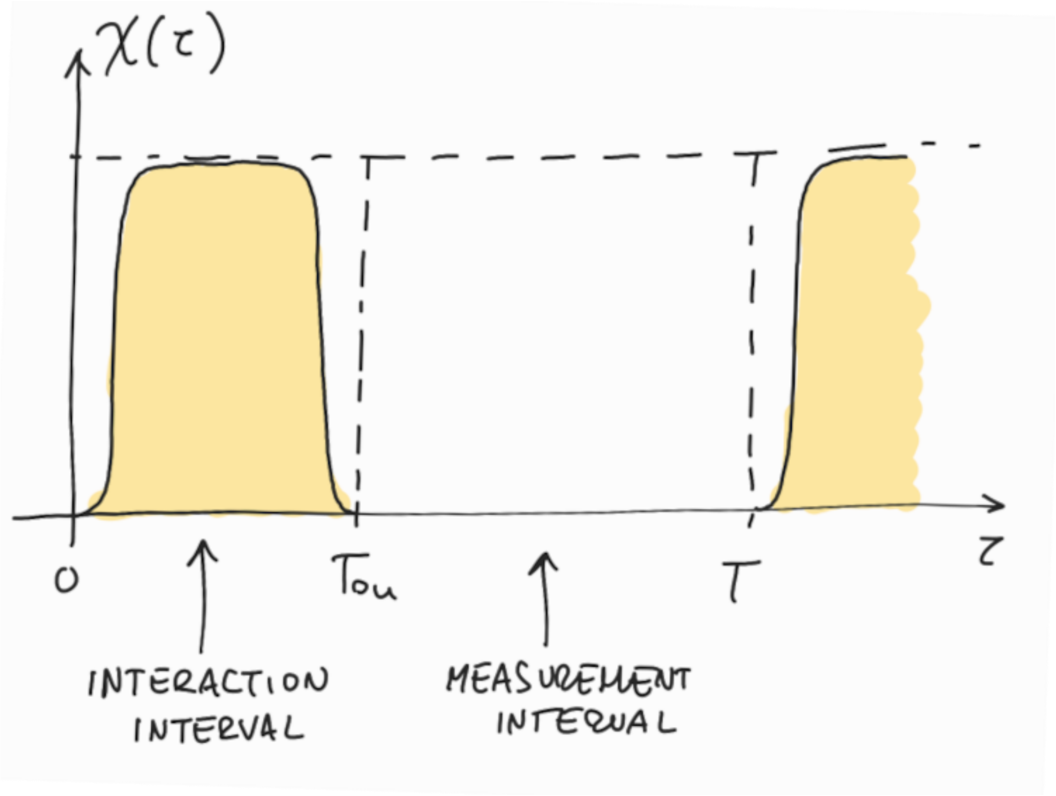
$$\hat{H}_{tot}(\tau) = \hat{H}_D\otimes\mathbb{I}_\phi + \mathbb{I}_\phi\otimes\hat{H}_\phi + \hat{H}_{int}(\tau)$$

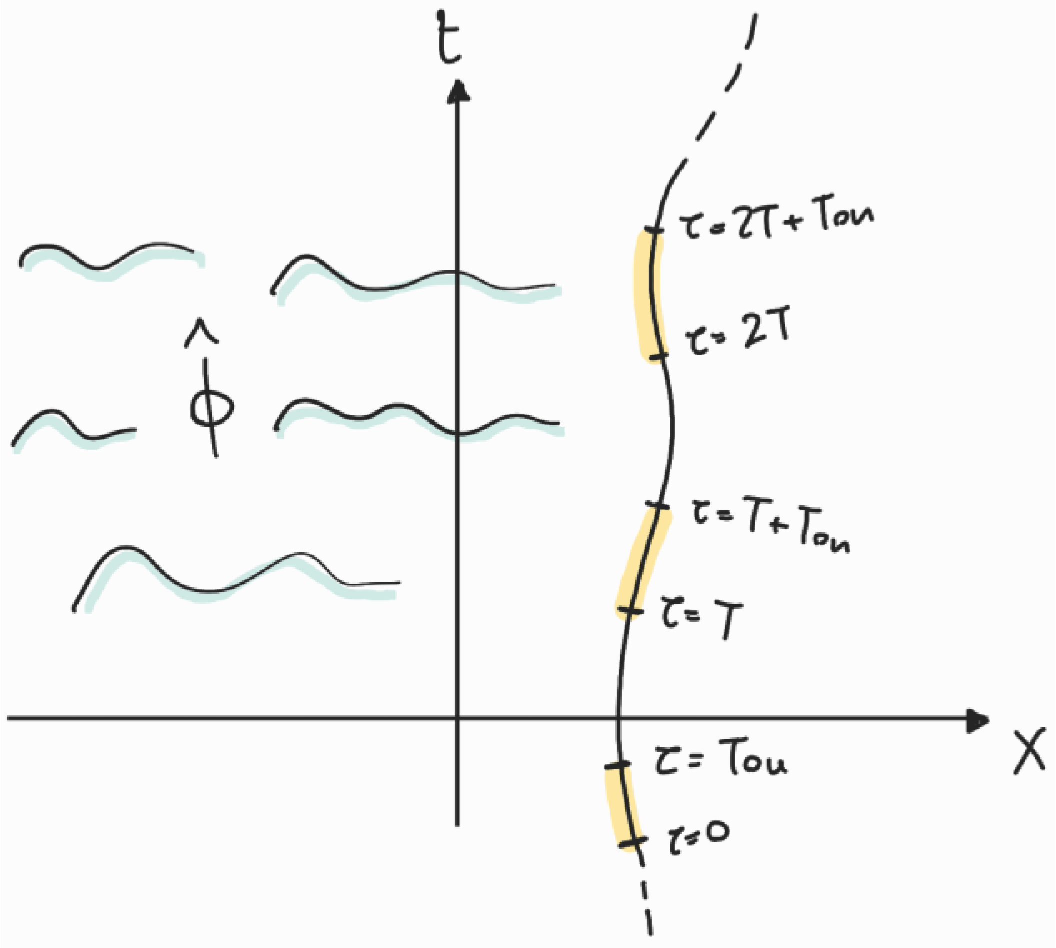
The switching function

$$\chi(\tau) = \sum_j \chi_j(\tau)\ , \quad supp\{\chi_j(\tau)\} \subseteq [jT, jT + T_{on}]$$



RM on UDW a detector

$$\chi(\tau) = \sum_i \chi_i(\tau)$$



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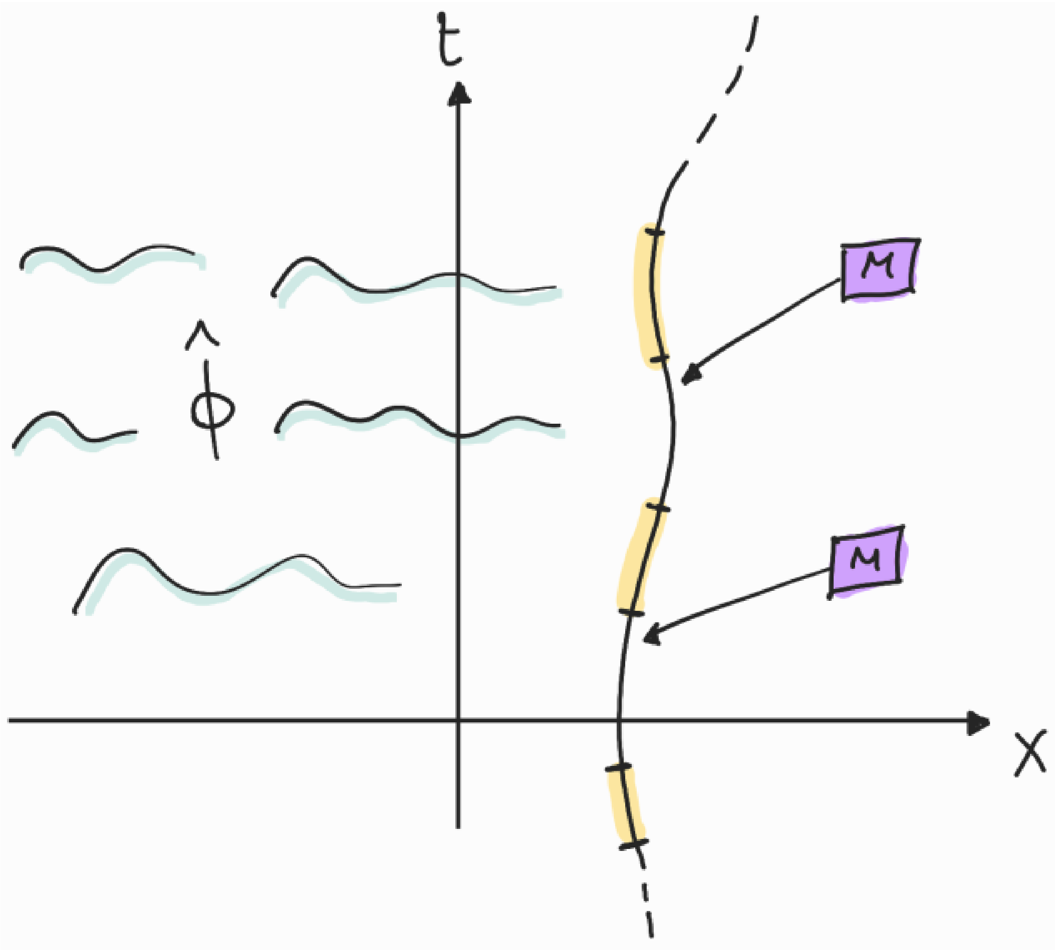
$$\begin{cases} \hat{M}_0 = |g\rangle\langle g| \\ \hat{M}_1 = |e\rangle\langle e| \end{cases}$$



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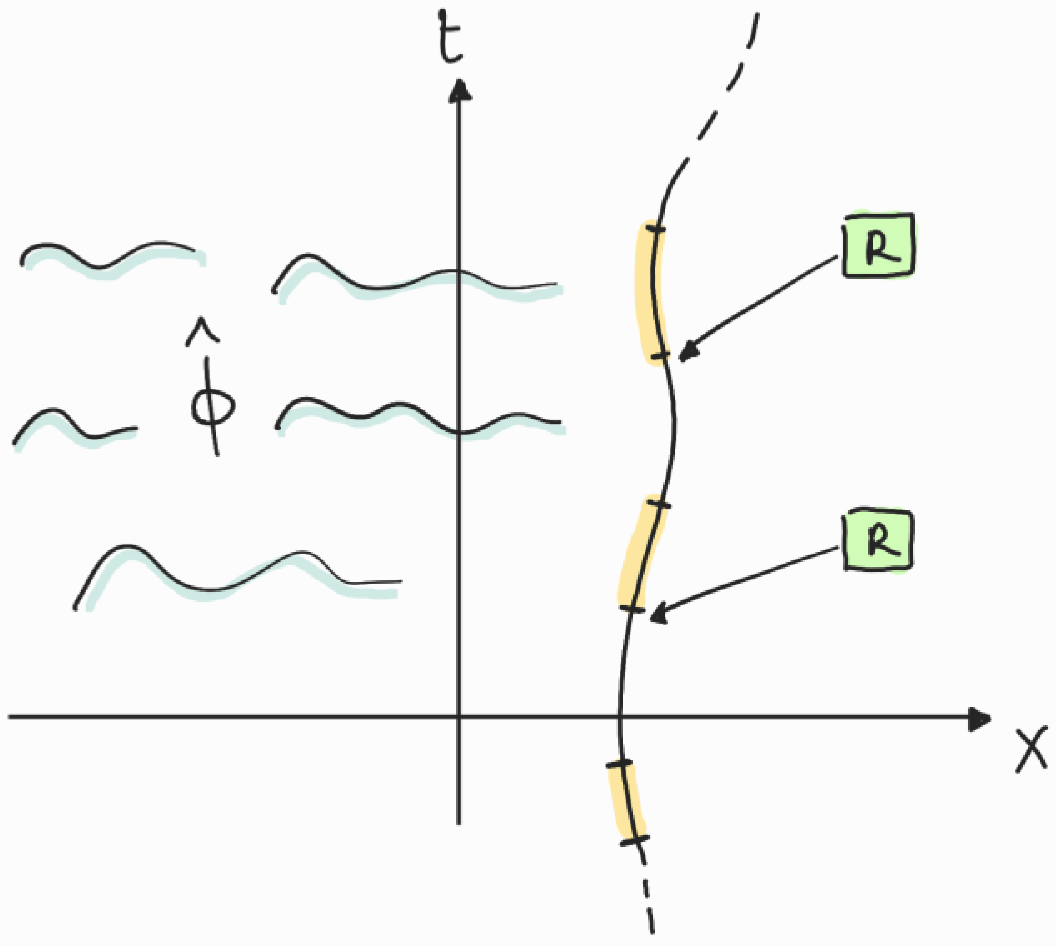

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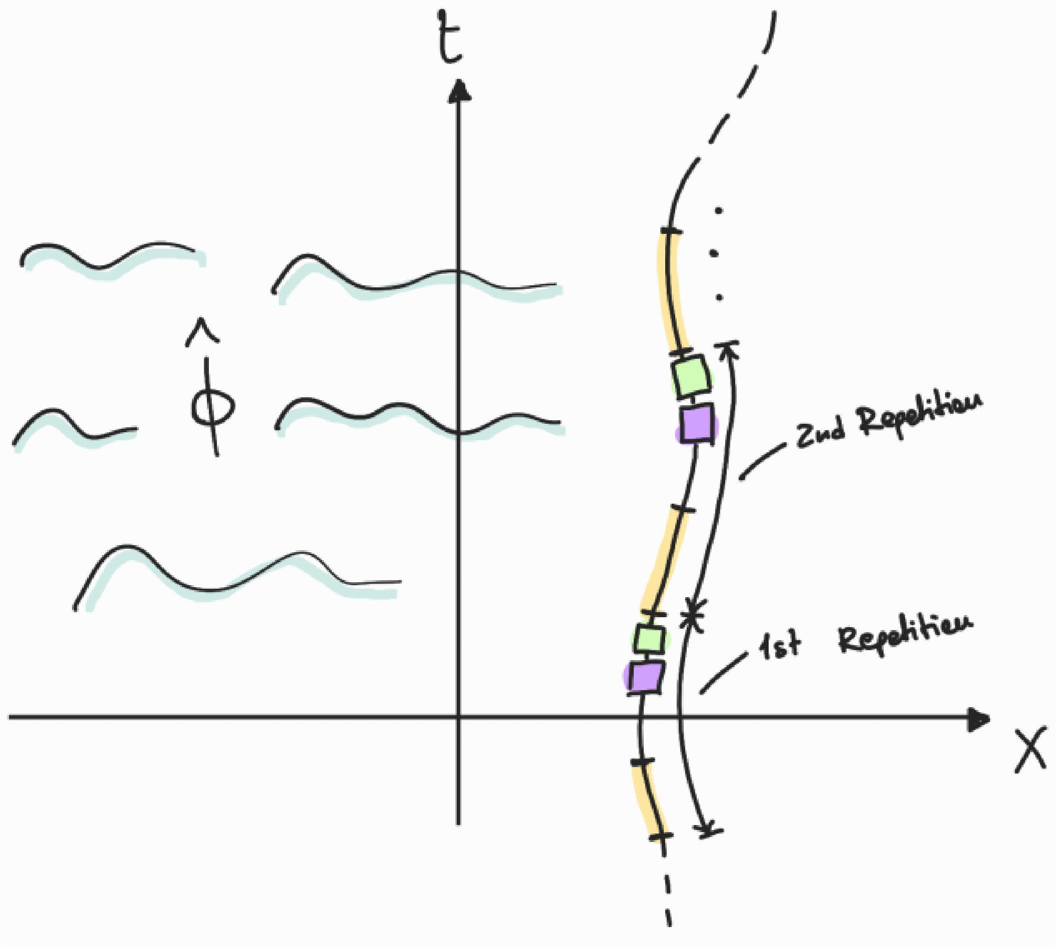

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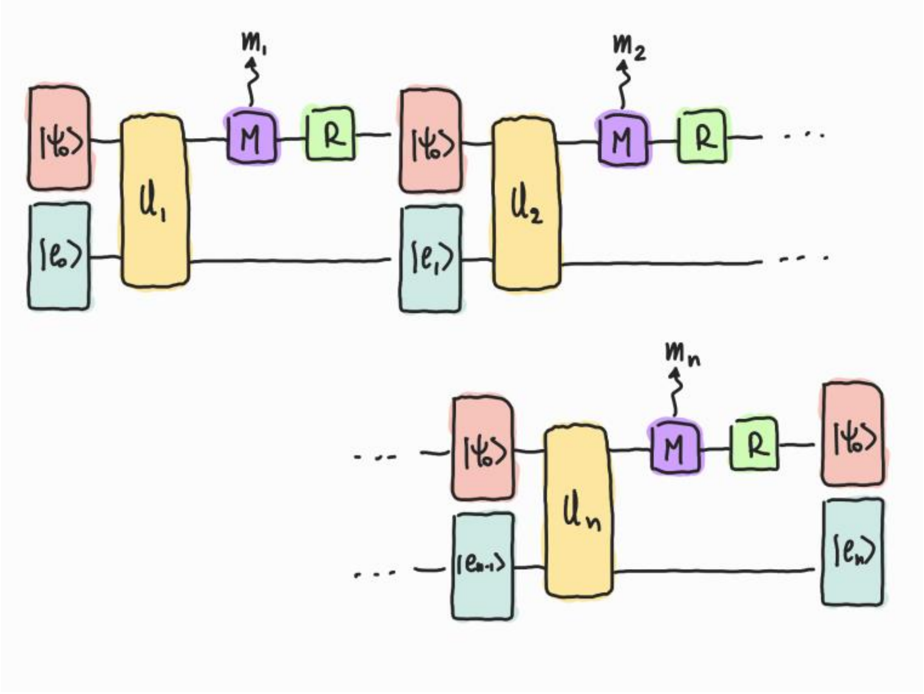

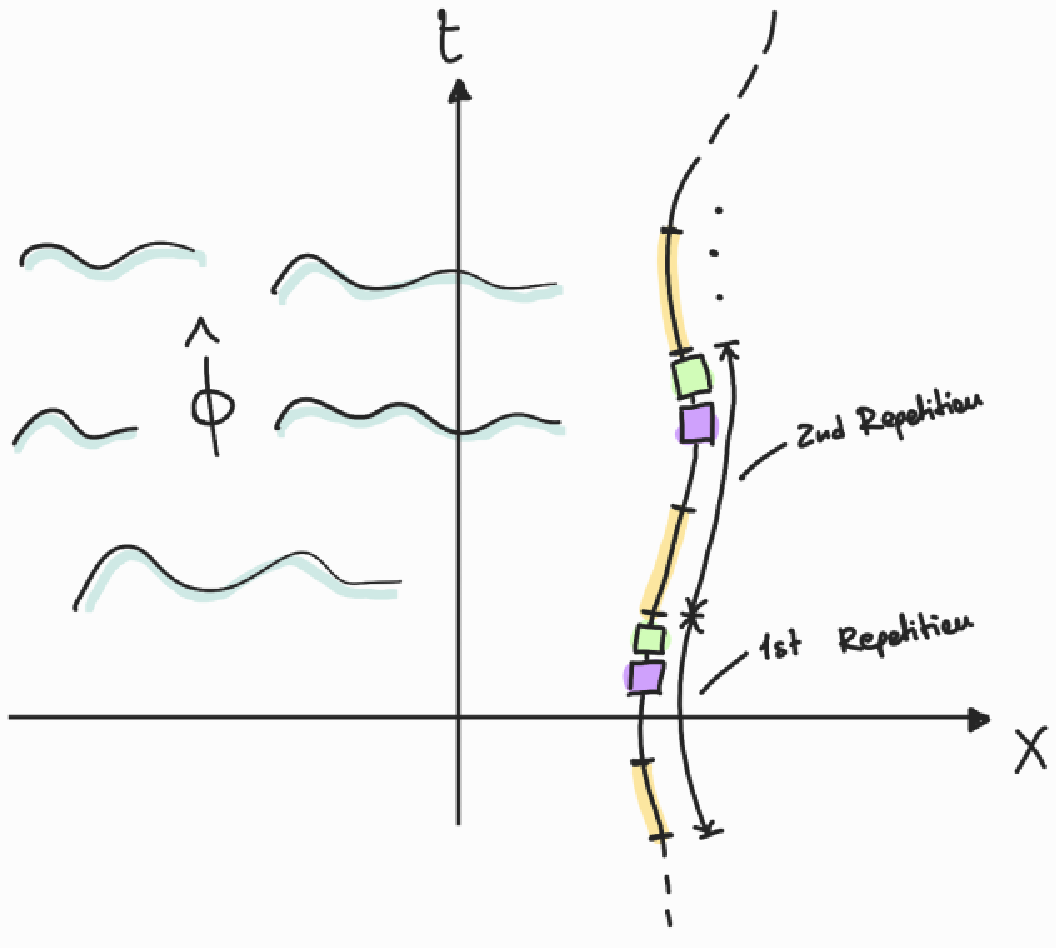

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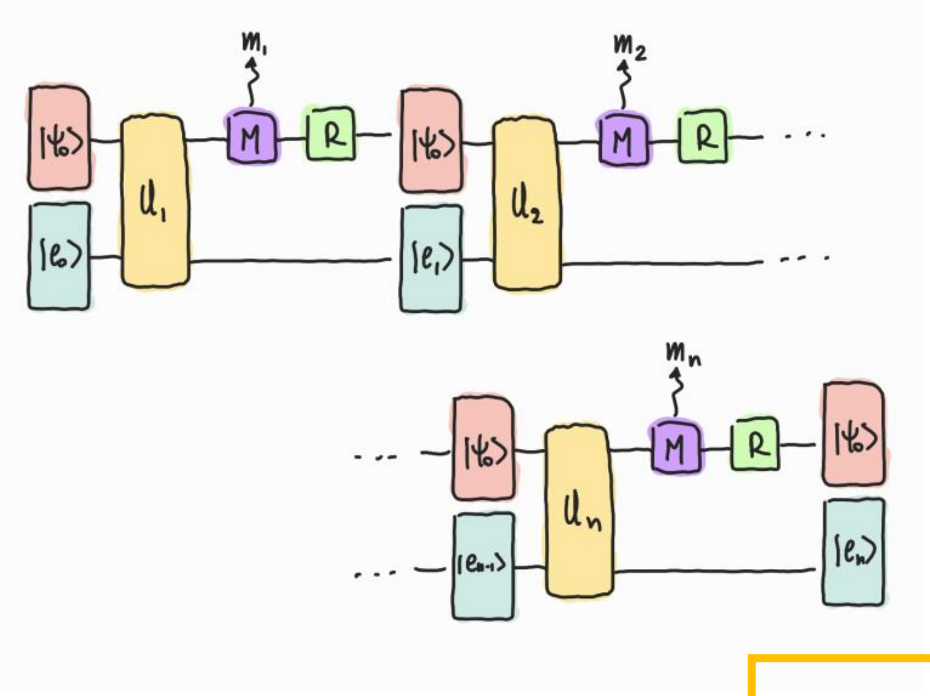
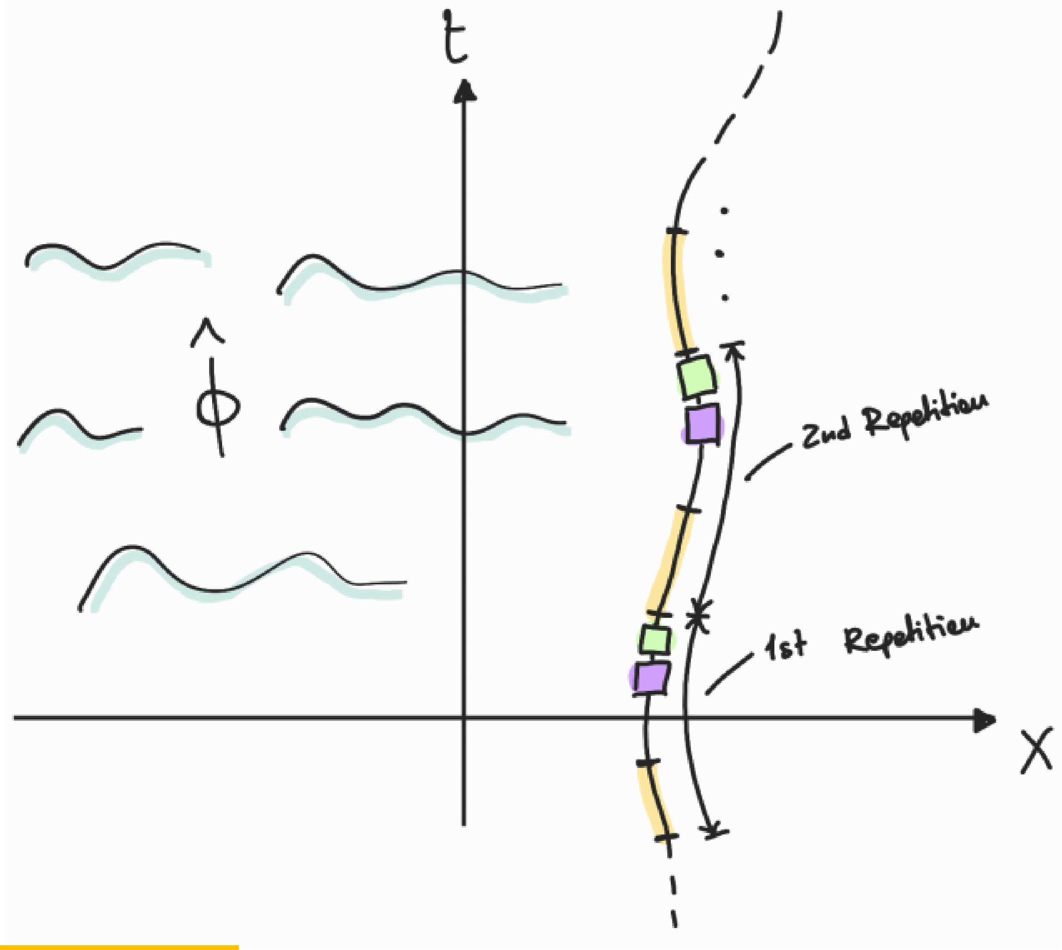




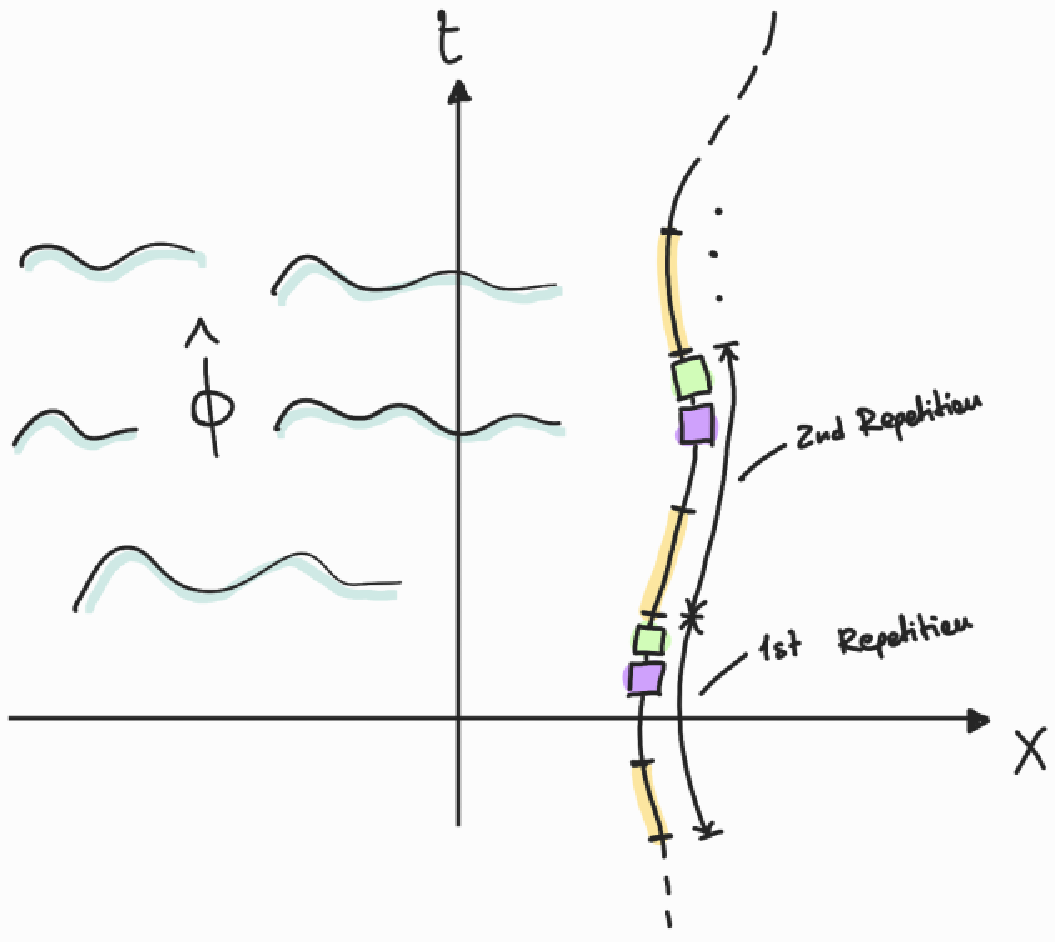
$$M_L = (m_1, \ldots, m_L)$$

Bit string Probabilities

$$M_L = (m_1, \dots, m_L)$$

$$M_L \iff (L; i_1, \dots, i_n)$$

$$(L; -) \equiv (0, \dots, 0)$$



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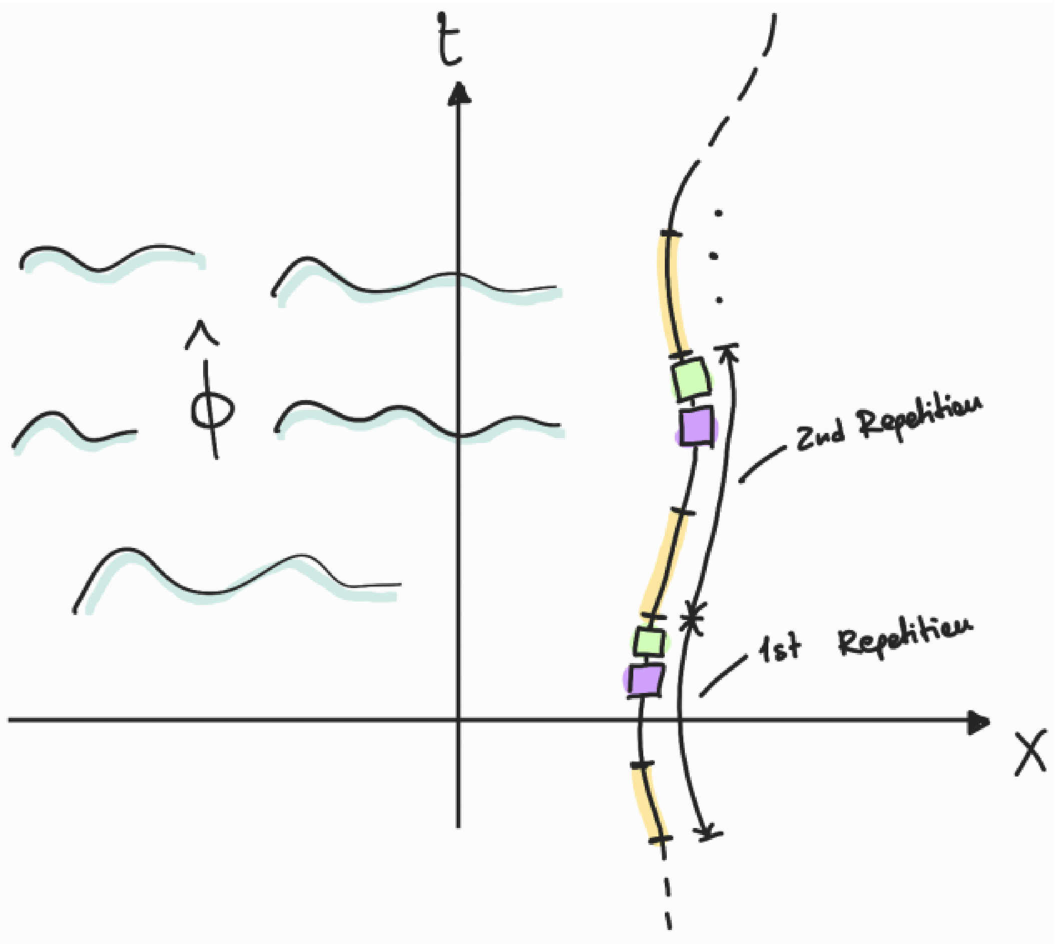

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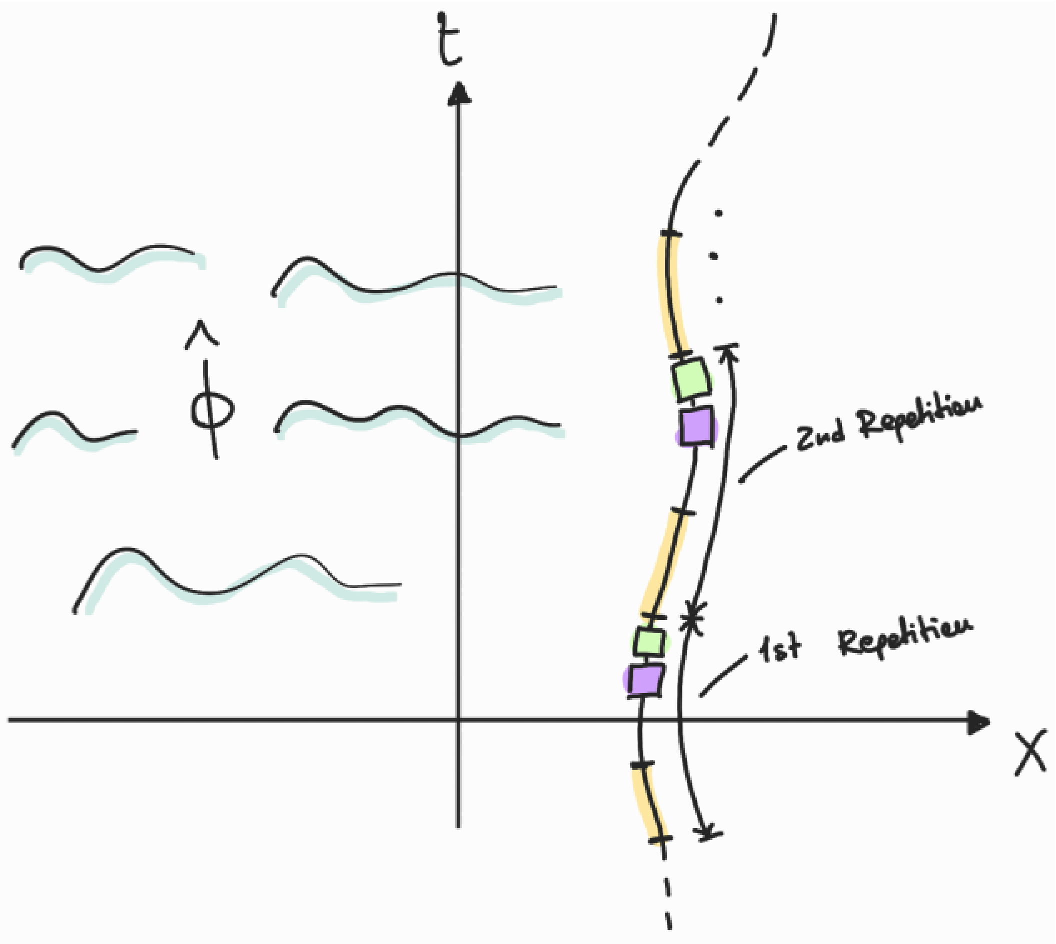

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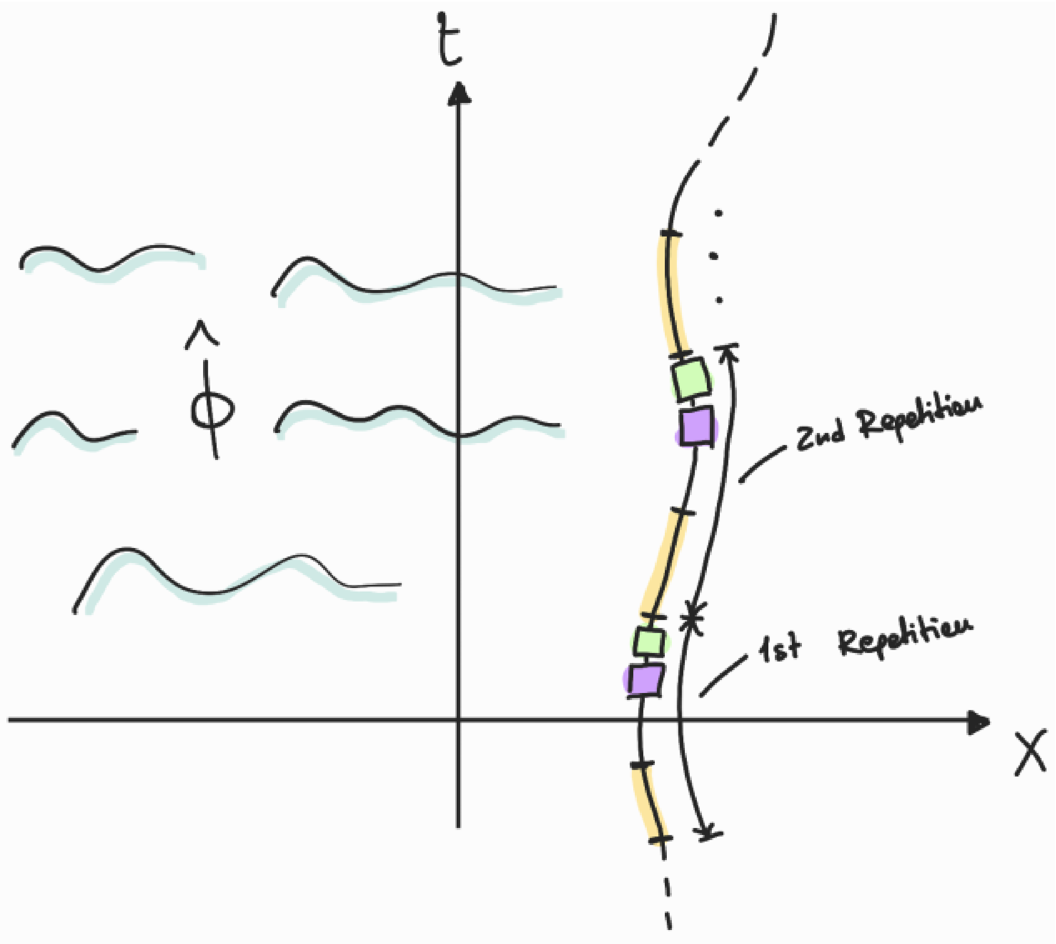

$$P[0|(L; i_1, \ldots, i_n)] \qquad P[1|(L; i_1, \ldots, i_n)]$$

1st transition

$$|g\rangle \otimes |0_M\rangle$$

1^{st} transition

$$|g\rangle \otimes |0_M\rangle - i\lambda \int_{LT}^{LT+T_{on}} d\tau \chi(\tau) \hat{m}(\tau) \, |g\rangle \otimes \hat{\phi}(X(\tau)) \, |0_M\rangle + O(\lambda^2) \sim |g\rangle \otimes |0_M\rangle + |e\rangle \otimes |\phi_1\rangle_L$$

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$$P[1|(L-1;-)] = \lambda^2\int_{LT}^{LT+T_{on}} d\tau\int_{LT}^{LT+T_{on}} d\tau'\chi(\tau)\chi(\tau')e^{-i\omega(\tau-\tau')}\mathcal{W}_2(X(\tau'),X(\tau))$$

$$\mathcal{W}_2(X(\tau'),X(\tau)) = \langle 0_M|\,\hat{\phi}(X(\tau'))\hat{\phi}(X(\tau))\,|0_M\rangle$$

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$$|g\rangle\otimes|0_M\rangle \xrightarrow[R]{} |g\rangle\otimes|\phi_1\rangle_L$$

2nd transition

$$|g\rangle \otimes |\phi_1\rangle_{i_1} - i\lambda \int_{LT}^{LT+T_{on}} d\tau \chi(\tau) \hat{m}(\tau) \, |g\rangle \otimes \hat{\phi}(X(\tau)) \, |\phi_1\rangle_{i_1} + O(\lambda^2) \sim |g\rangle \otimes |0_M\rangle + |e\rangle \otimes |\phi_2\rangle_L$$

2^{nd} transition

$$|g\rangle\otimes|\phi_1\rangle_{i_1} - i\lambda\int_{LT}^{LT+T_{on}} d\tau\chi(\tau)\hat{m}(\tau)\,|g\rangle\otimes\hat{\phi}(X(\tau))\,|\phi_1\rangle_{i_1} + O(\lambda^2) \sim |g\rangle\otimes|0_M\rangle + |e\rangle\otimes|\phi_2\rangle_L$$

$$\begin{aligned} P[1|(L-1;i_1)] &= \frac{\lambda^4}{q}\iint_L \langle\phi_1|_L\,\hat{\phi}(X(\tau_L'))\hat{\phi}(X(\tau_L))\,|\phi_1\rangle_L \\ &= \frac{\lambda^4}{q}\iint_L\iint_{i_1} \mathcal{W}_4(X(\tau_2'),X(\tau_1'),X(\tau_1)X(\tau_2)) \end{aligned}$$

- $\int_{N_i} = 2\int_{TN_i}^{TN_i+T_{on}} du_i \int_0^{u_i-TN_i} ds_i\chi(u_i)\chi(u_i-s_i)\cos(\omega s_i)$
- $\mathcal{W}_4(X(\tau_2'),X(\tau_1'),X(\tau_1)X(\tau_2)) = \langle 0_M|\,\hat{\phi}(X(\tau_{i_1}'))\hat{\phi}(X(\tau_L'))\hat{\phi}(X(\tau_L))\hat{\phi}(X(\tau_{i_1}))\,|0_M\rangle$

2^{nd} transition

$$|g\rangle\otimes|\phi_1\rangle_{i_1} - i\lambda\int_{LT}^{LT+T_{on}} d\tau\chi(\tau)\hat{m}(\tau)\,|g\rangle\otimes\hat{\phi}(X(\tau))\,|\phi_1\rangle_{i_1} + O(\lambda^2) \sim |g\rangle\otimes|0_M\rangle + |e\rangle\otimes|\phi_2\rangle_L$$

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- $\mathcal{W}_4(X(\tau_2'),X(\tau_1'),X(\tau_1)X(\tau_2)) = \langle 0_M|\,\hat{\phi}(X(\tau_{i_1}'))\hat{\phi}(X(\tau_L'))\hat{\phi}(X(\tau_L))\hat{\phi}(X(\tau_{i_1}))\,|0_M\rangle$

$$|\phi_1\rangle_{i_1} \to |\phi_2\rangle_L$$

n^{th} transition

$$P[1|(L-1;i_1,\dots,i_{n-1})] =$$

$$= \frac{\lambda^{2n}}{q \times q(2;i_2)\dots q(n-1;i_{n-1})} \iint_L \prod_{j=1}^{n-1} \iint_j \langle\phi_{n-1}|_L\, \hat{\phi}(X(\tau_L'))\hat{\phi}(X(\tau_L))\,|\phi_{i_{n-1}}\rangle_L$$

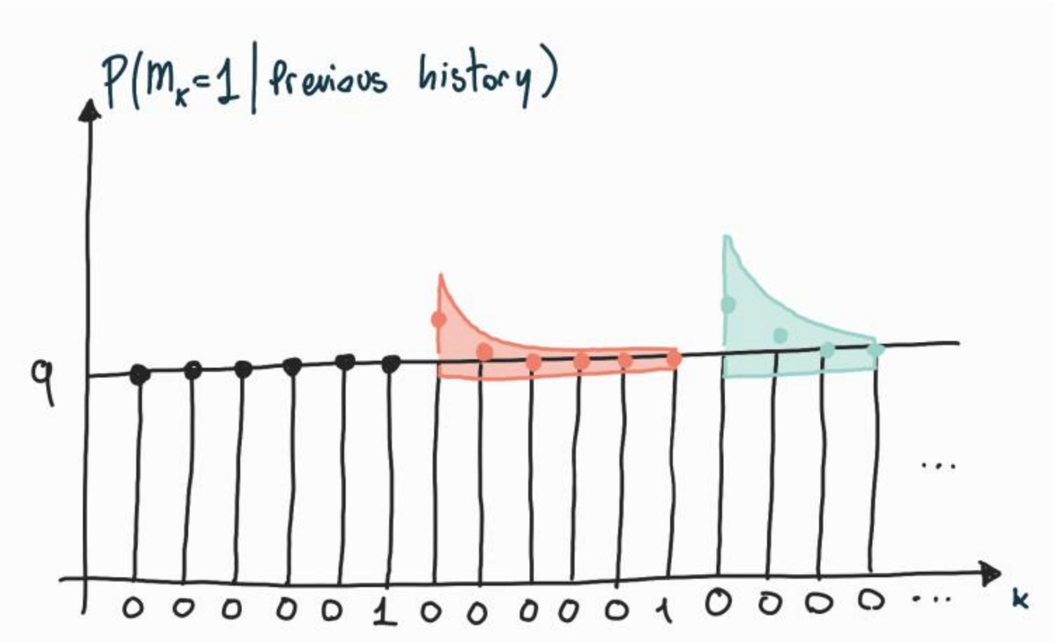
$$= \frac{\lambda^{2n}}{q \times q(2;i_2)\dots q(n-1;i_{n-1})} \iint_L \prod_{j=1}^{n-1} \iint_j \mathcal{W}_{2n}(X(\tau_L'), X(\tau_L), \dots)$$

Upper and Lower Bounds

Using the **strong Huygens principle**, and under the assumption that

$$T_{on}\omega \leq \frac{\pi}{2} \qquad\qquad T_{on} \ll T_{off}$$

One finds



$$\begin{cases} q \leq P[1|(L;i_1)] \leq q(1+2\gamma^2) \\ \dfrac{q}{(1+2\gamma^2)} \leq P[1|(L;i_1,i_2)] \leq q(1+6\gamma^2+8\gamma^3) \\ \dots \\ \dfrac{q}{(1+\dots+\frac{(2n-3)!!}{\sqrt{e}}\gamma^{n-1})} \leq P[1|(L;i_1,\dots,i_{n-1})] \leq q(1+\dots+\dfrac{(2n-1)!!}{\sqrt{e}}\gamma^n) \end{cases}$$

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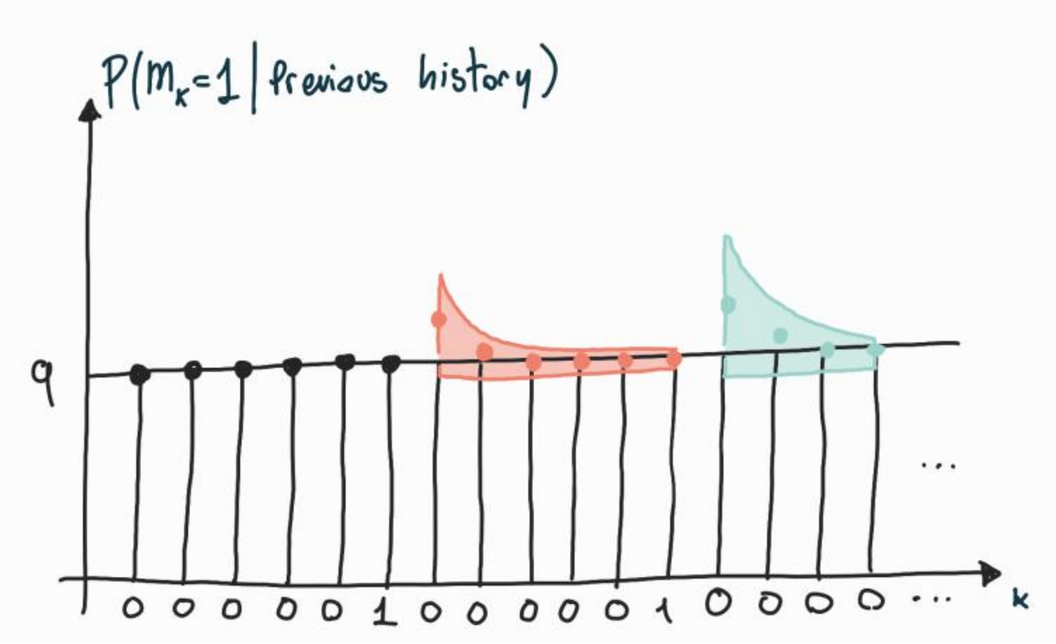

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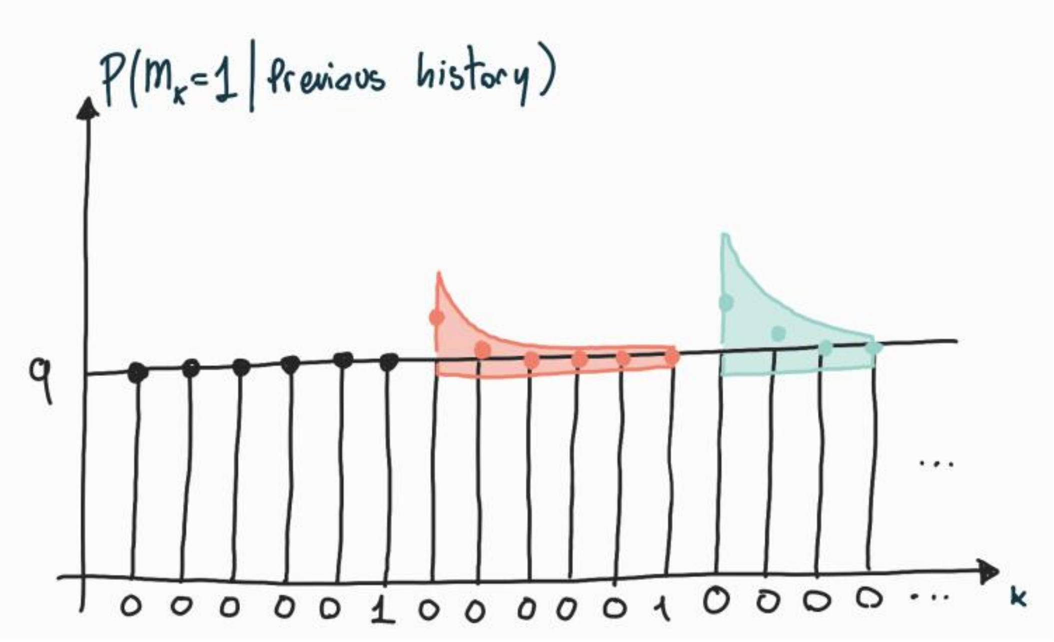

$$p[1|(L;i_1,\dots,i_n)] \simeq q$$

$$p(M_L) \simeq q \times q^n(1-q)^{L-n}$$

Conclusions

- In some cases, even if holding the Born rule has no predicting power
- In weak RM one recovers an almost-Born rule
- FAPP, UDW detectors see strings consistent with asymptotic populations (for any trajectory) even in the RM scenario
- ...and more!

Thank you!