Virtual Linear Map Algorithm

for classical boost in near-term quantum computing

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arXiv:2207.01360
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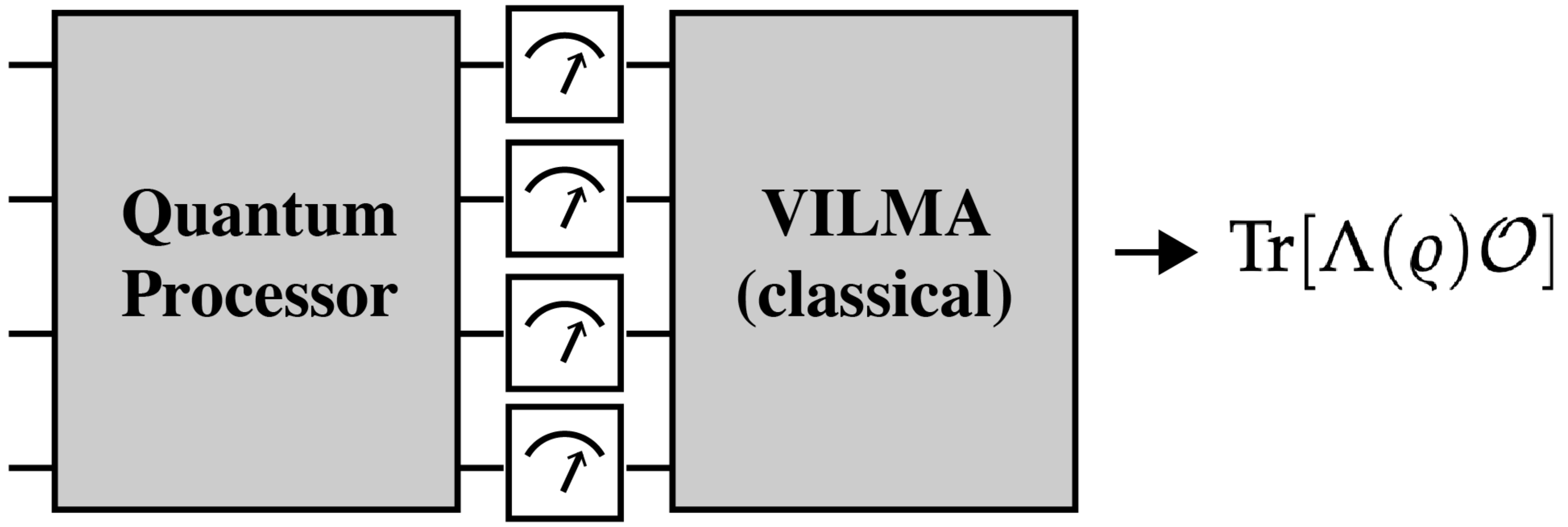
The VILMA method

overview

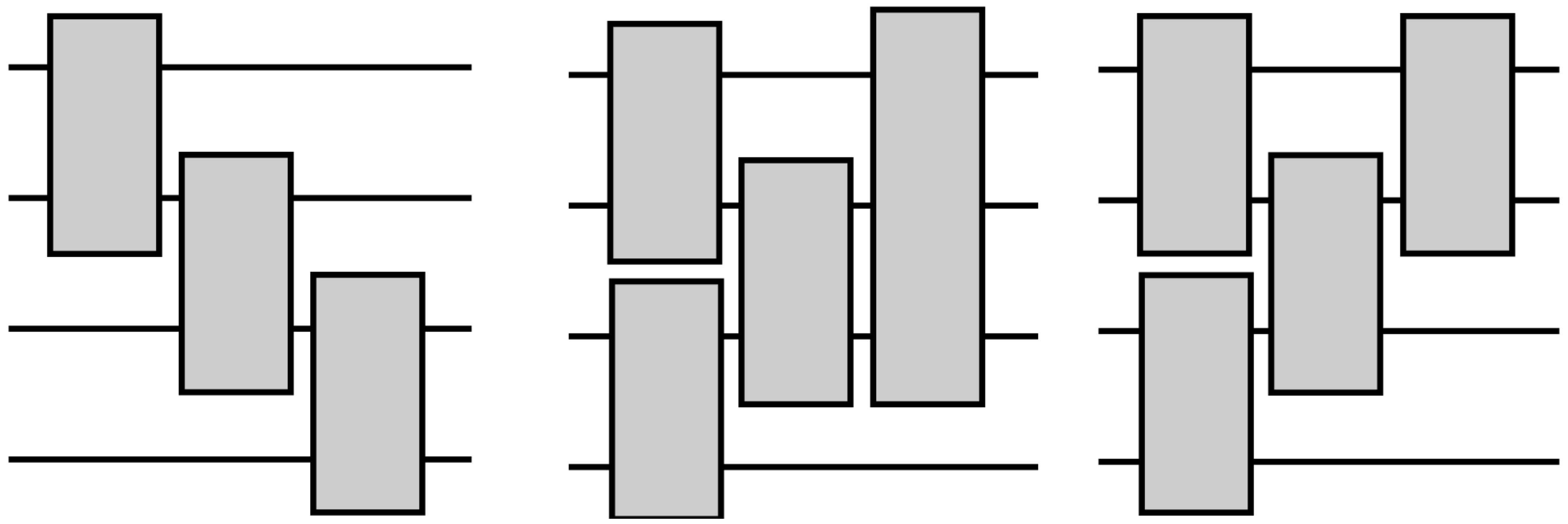
A)



B)

C)

D)

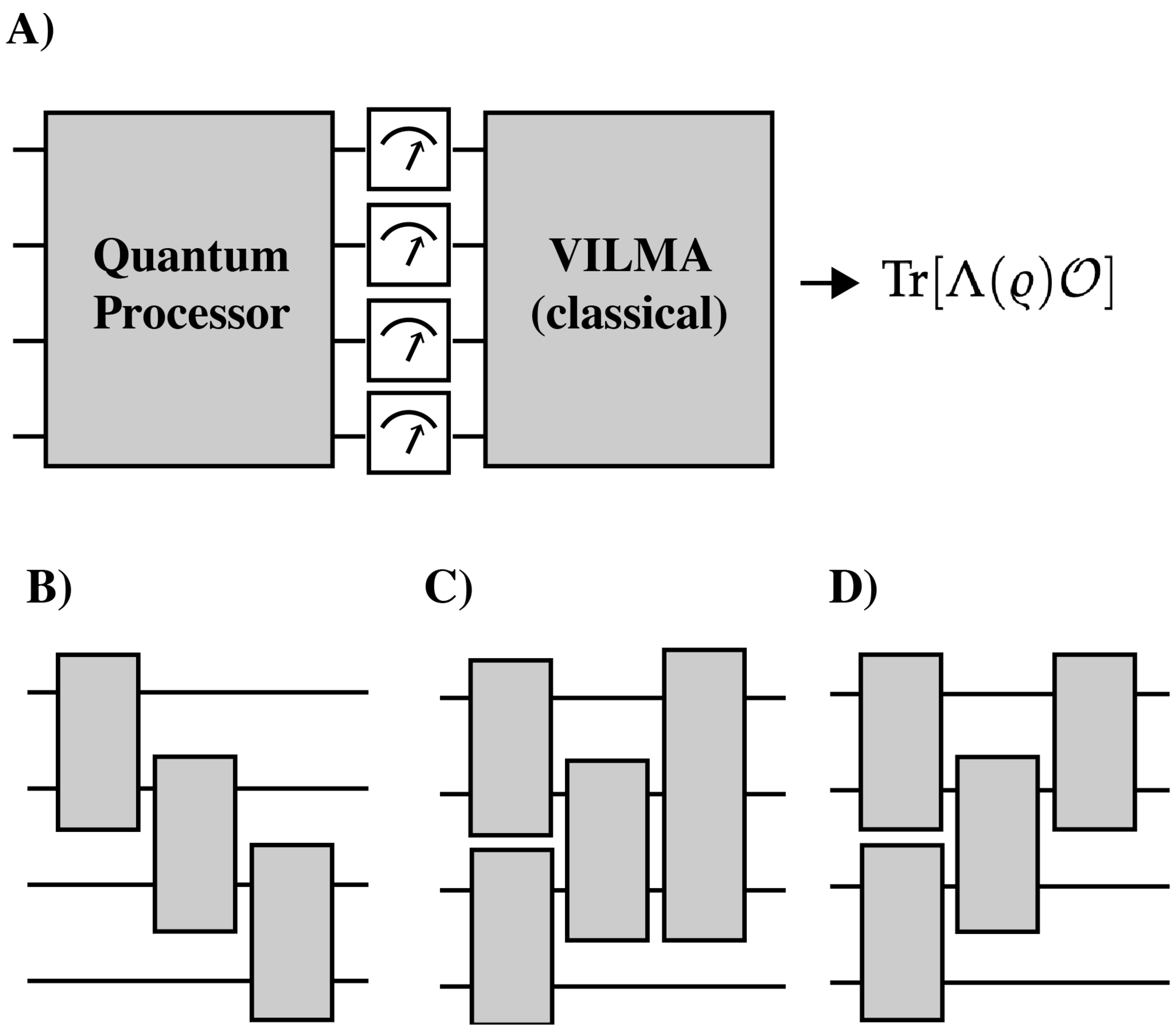


Transforming states in post-processing

- The goal is to compute expectation values of observables on the image of the physical state through some map
- We do NOT want to do full state tomography (unfeasible for large systems)
- We want the map to be circuit-like (useful for quantum computing)

The VILMA method

operator averages on transformed states



Using IC measurement data

- Consider an IC-POVM with effects

$$\{\Pi_{\mathbf{m}} = \bigotimes_{i=1}^{N} \Pi_{m_i}^{(i)}\}$$

- It defines a set of dual effects such that

$$\mathcal{O} = \sum_{\mathbf{m}} \mathrm{Tr}[\mathcal{O}\Pi_{\mathbf{m}}] D_{\mathbf{m}} \text{ for all } \mathcal{O}$$

- The state of the QPU reads $\rho = \sum_{\mathbf{m}} p_{\mathbf{m}} D_{\mathbf{m}}$, with

$$D_{\mathbf{m}} = \bigotimes_{i=1}^{N} D_{m_i}^{(i)} \text{ and } p_{\mathbf{m}} = \mathrm{Tr}[\rho \Pi_{\mathbf{m}}]$$

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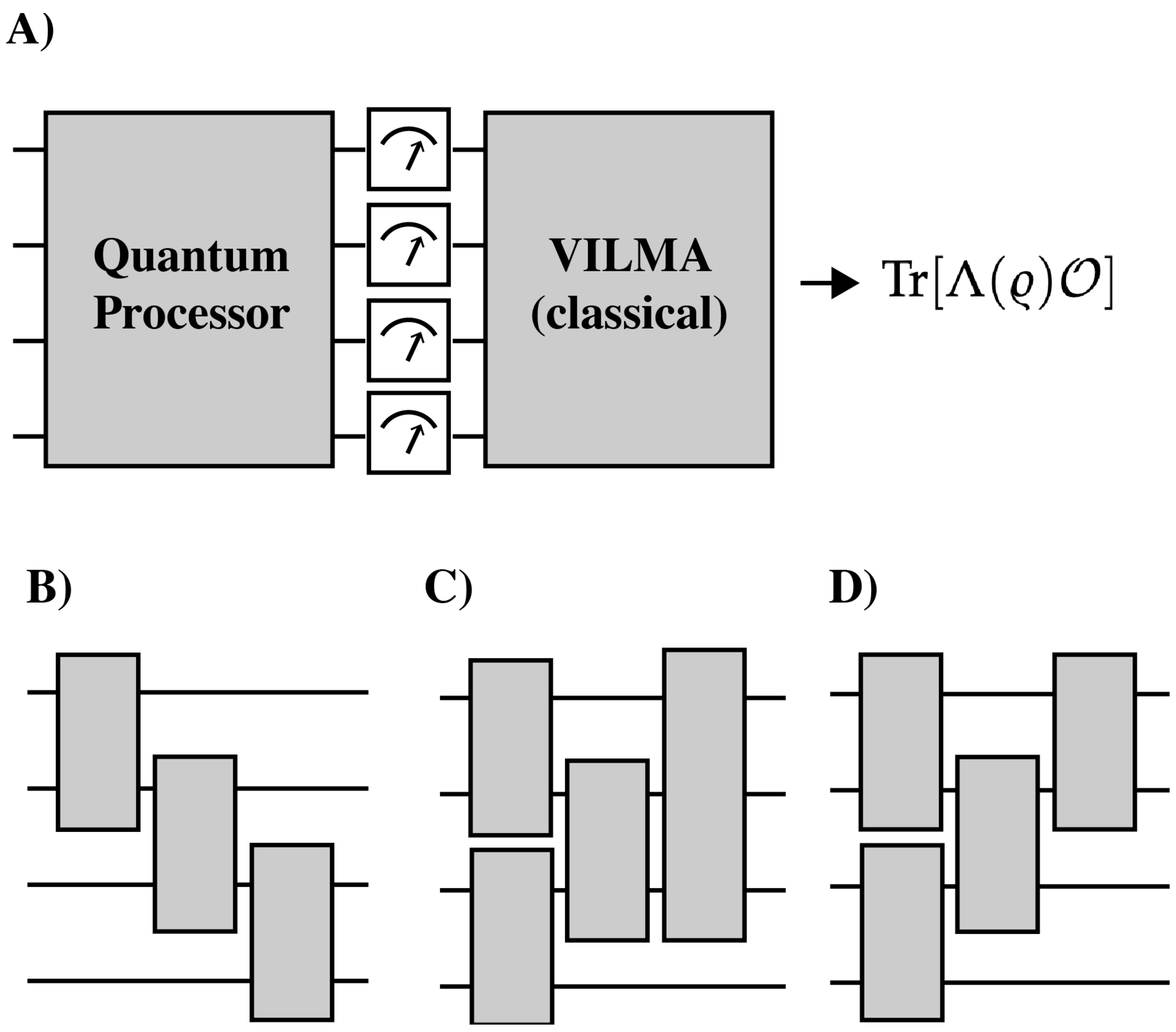

Using IC measurement data

- Let the observable be $\mathcal{O} = \sum_{\mathbf{k}} c_{\mathbf{k}} P_{\mathbf{k}}$
- For finitely many samples S, $\rho_S = \sum_{i=1}^{S} D_{\mathbf{m}_i}/S$,

  fulfilling $\lim_{S\to\infty} \rho_S = \rho$
- Since $\mathrm{Tr}[\Lambda(\rho)\mathcal{O}] = \lim_{S\to\infty}\mathrm{Tr}[\Lambda(\rho_S)\mathcal{O}]$,

  $$\lim_{S\to\infty} \sum_{i=1}^{S} \frac{1}{S} \sum_{\mathbf{k}} c_{\mathbf{k}} \mathrm{Tr}[\Lambda(D_{\mathbf{m}_i})P_{\mathbf{k}}]$$

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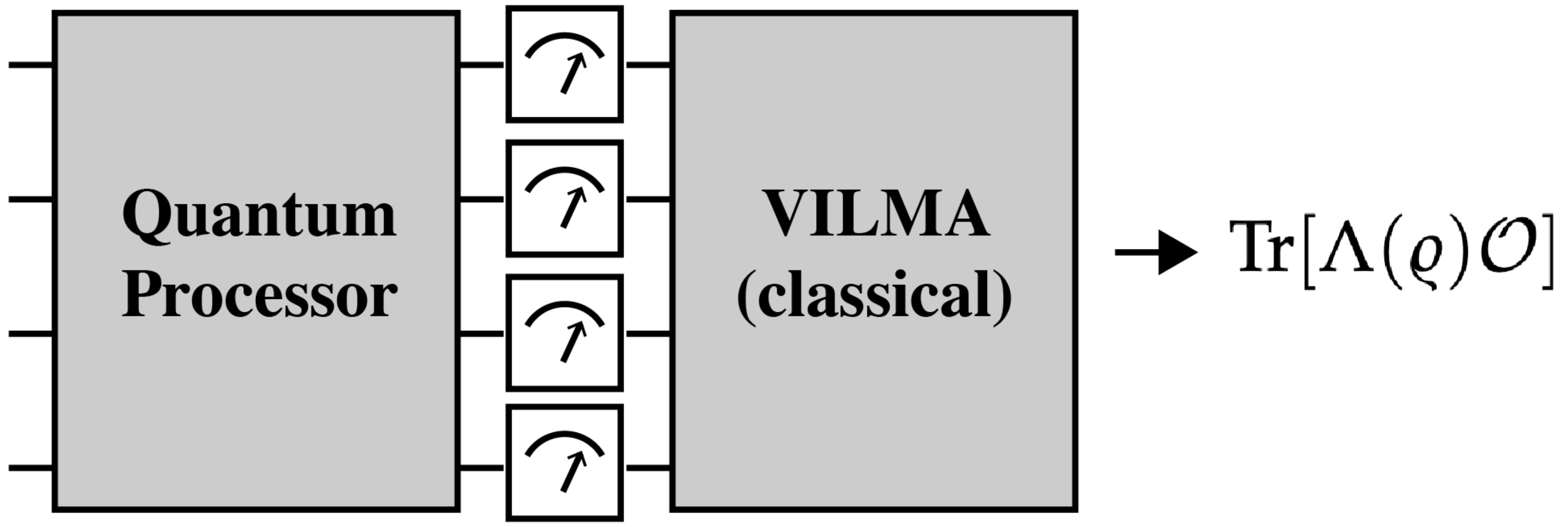

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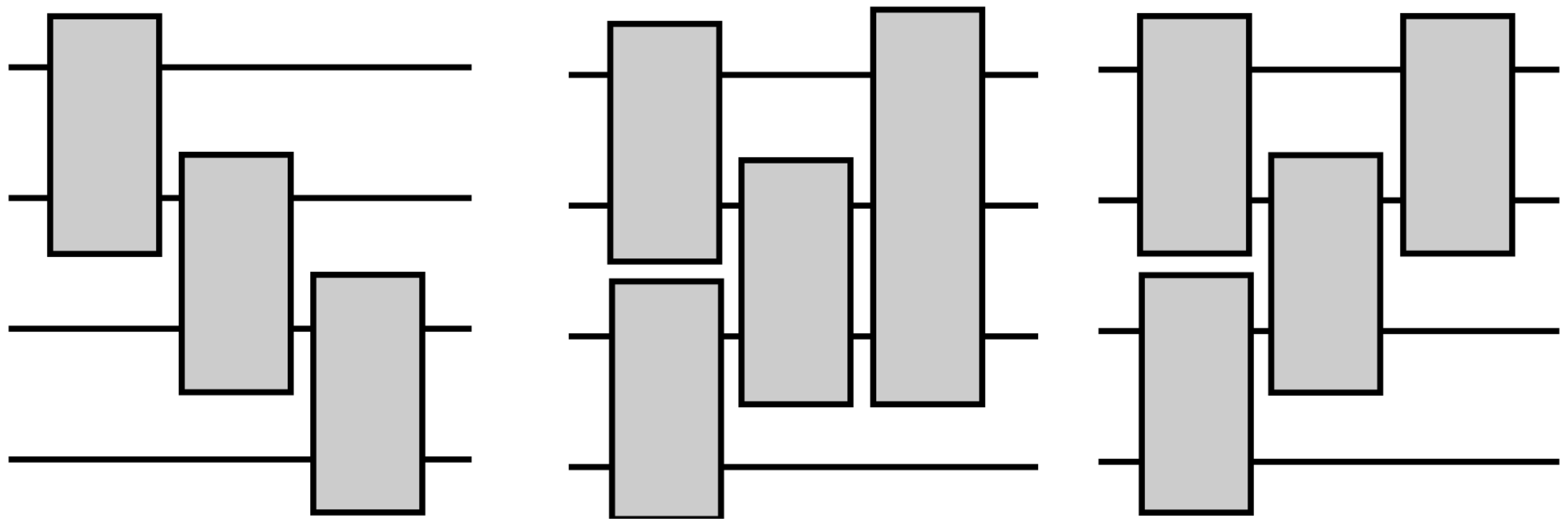

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- Since $\mathrm{Tr}[\Lambda(\rho)\mathcal{O}] = \lim_{S\to\infty} \mathrm{Tr}[\Lambda(\rho_S)\mathcal{O}]$,

$$\lim_{S\to\infty} \sum_{i=1}^{S} \frac{1}{S} \boxed{\sum_{\mathbf{k}} c_{\mathbf{k}} \mathrm{Tr}[\Lambda(D_{\mathbf{m}_i})P_{\mathbf{k}}]} \longleftarrow \omega_{\mathbf{m}_i}$$

- The estimator $\bar{\mathcal{O}}_\Lambda = \sum_{i=1}^{S} \omega_{\mathbf{m}_i}/S$ is unbiased

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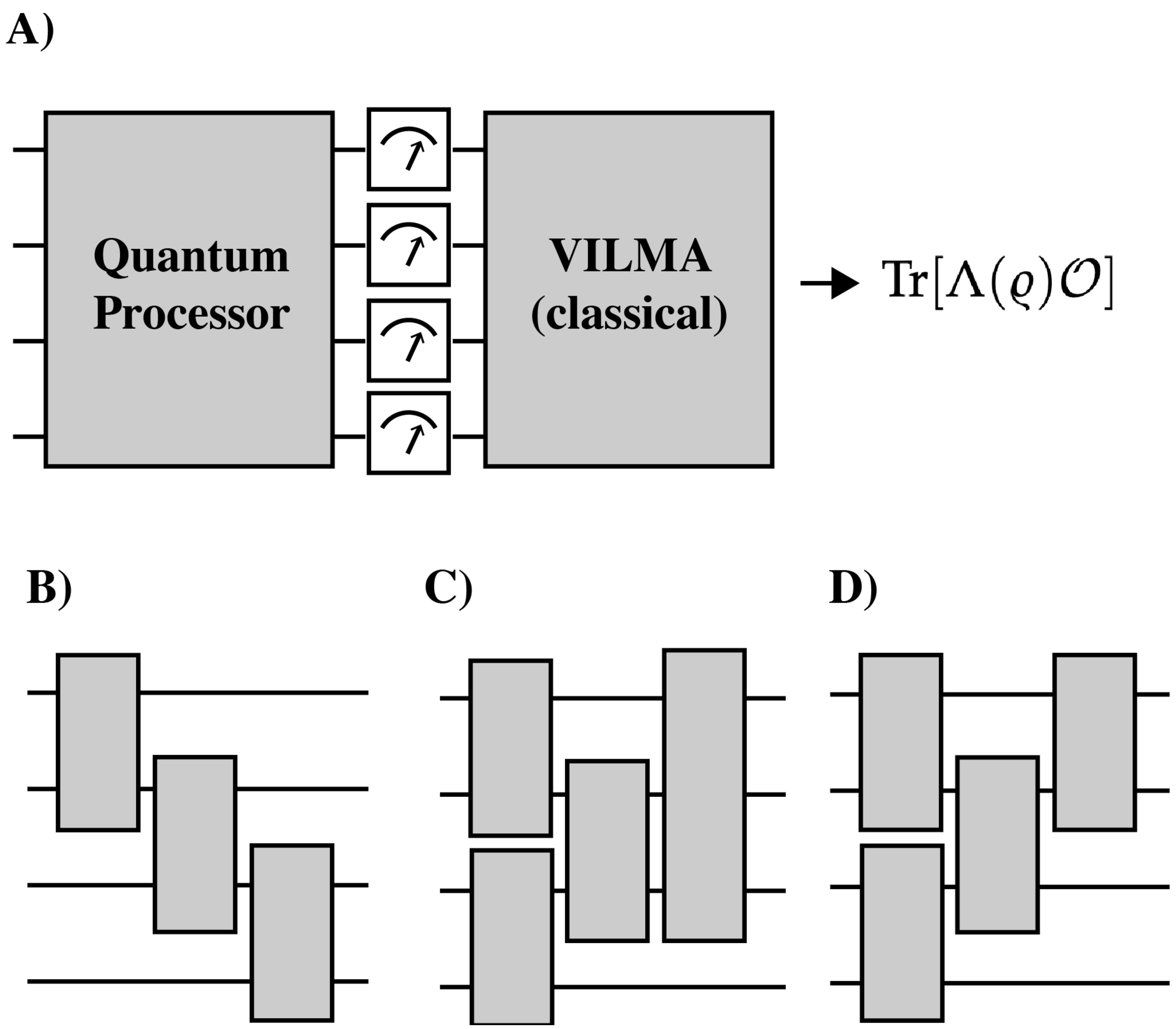

Using IC measurement data

- In addition to the estimation of the mean $\bar{\mathcal{O}}_\Lambda$, we can estimate the statistical error of the estimation,

$$\sigma = \sqrt{\bar{V}(\mathcal{O}_\Lambda)/S},$$

where $\bar{V}(\mathcal{O}_\Lambda) \equiv [\sum_{i=1}^{S} \omega_{\mathbf{m}_i}^2/S - (\bar{\mathcal{O}}_\Lambda)^2]S/(S-1)$ is

an unbiased estimator of the variance of $\omega_{\mathbf{m}_i}$

The VILMA method

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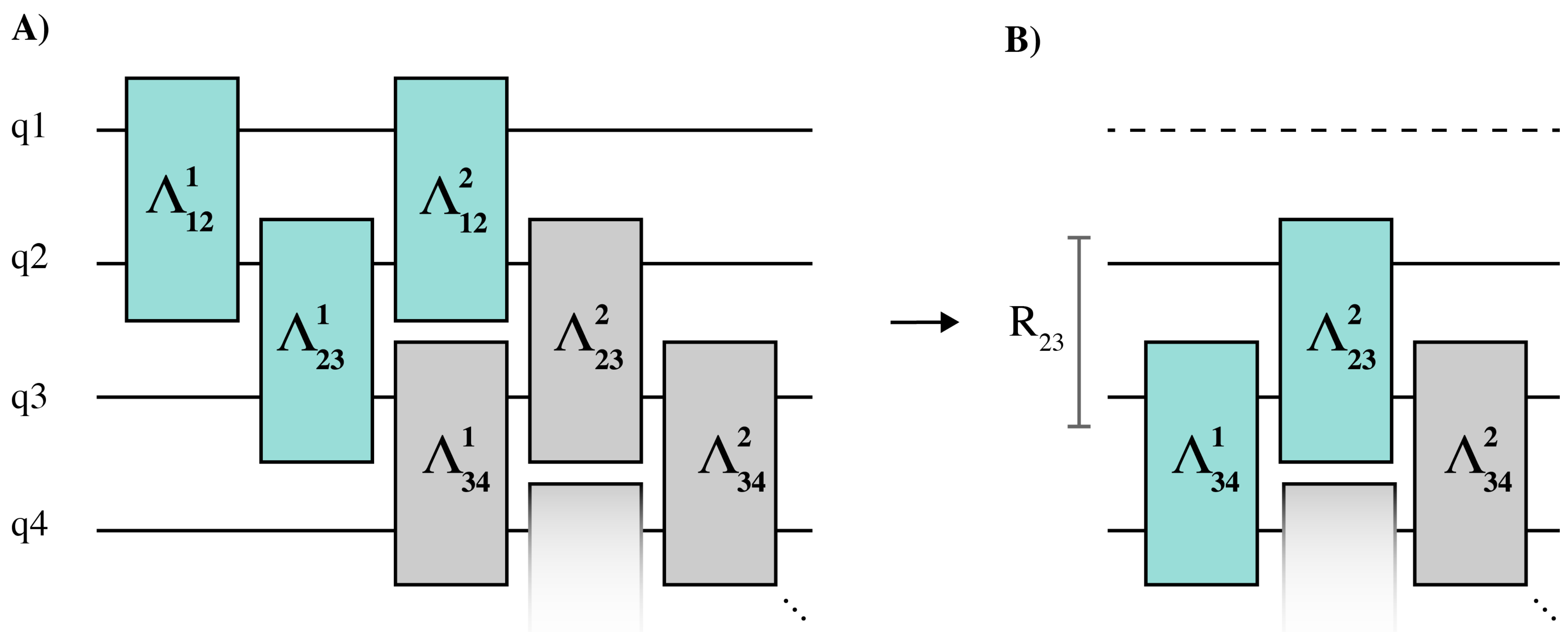

Traces involving mapped dual effects

- Computing the traces $\mathrm{Tr}[\Lambda(D_{\mathbf{m}_i})P_{\mathbf{k}}]$ in $\omega_{\mathbf{m}_i}$ can be challenging given the dimension of $\Lambda(D_{\mathbf{m}_i})$

- We exploit the causal cone structure of the circuit to bypass explicit high-dimensional reconstructions

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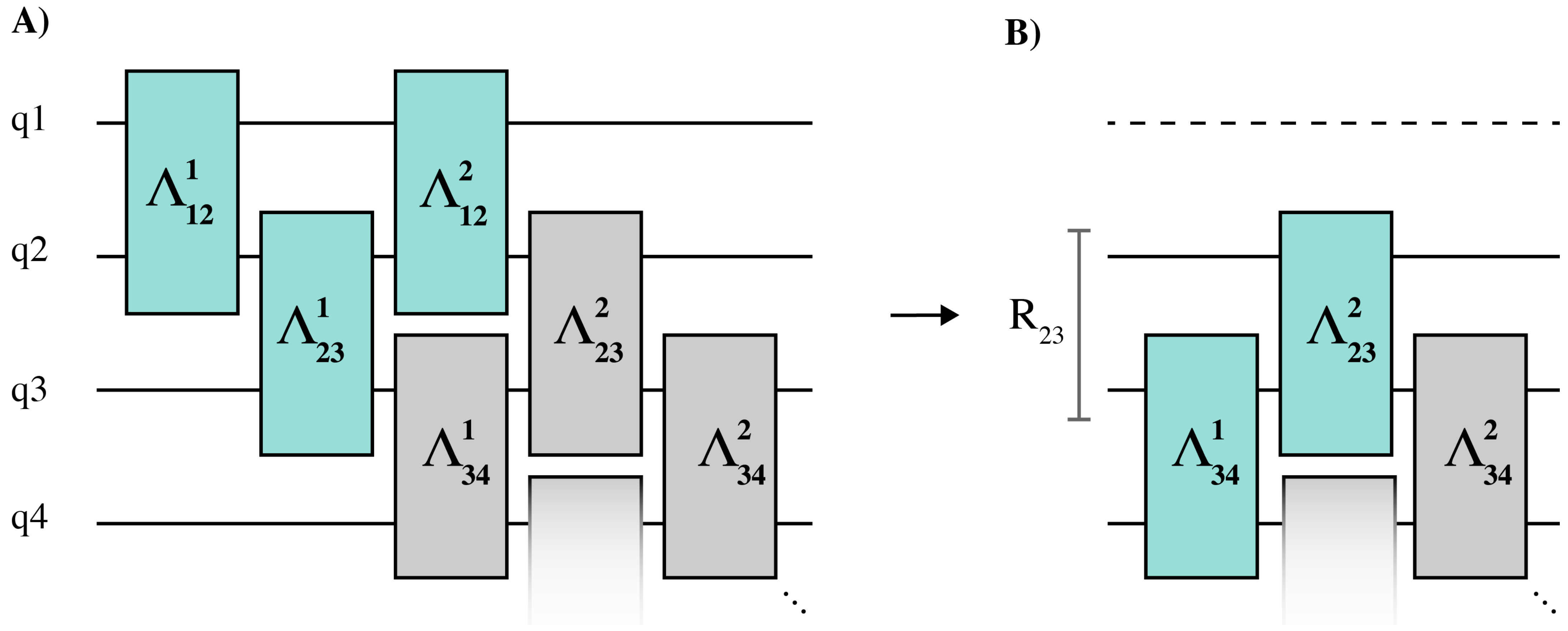

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- Compute $\Lambda^2_{12} \circ \Lambda^1_{23} \circ \Lambda^1_{12}(D_{m_1} \otimes D_{m_2} \otimes D_{m_3})$
- $R_{2,3} = \mathrm{Tr}_1[\Lambda^2_{12} \circ \Lambda^1_{23} \circ \Lambda^1_{12}(D_{m_1} \otimes D_{m_2} \otimes D_{m_3})P_{k_1} \otimes \mathbb{I}_2 \otimes \mathbb{I}_3]$
- Compute $\Lambda^2_{23} \circ \Lambda^1_{34}(R_{2,3} \otimes D_{m_4})$
- $R_{3,4} = \mathrm{Tr}_2[\Lambda^2_{23} \circ \Lambda^1_{34}(R_{2,3} \otimes D_{m_4})P_{k_2} \otimes \mathbb{I}_3 \otimes \mathbb{I}_4]$

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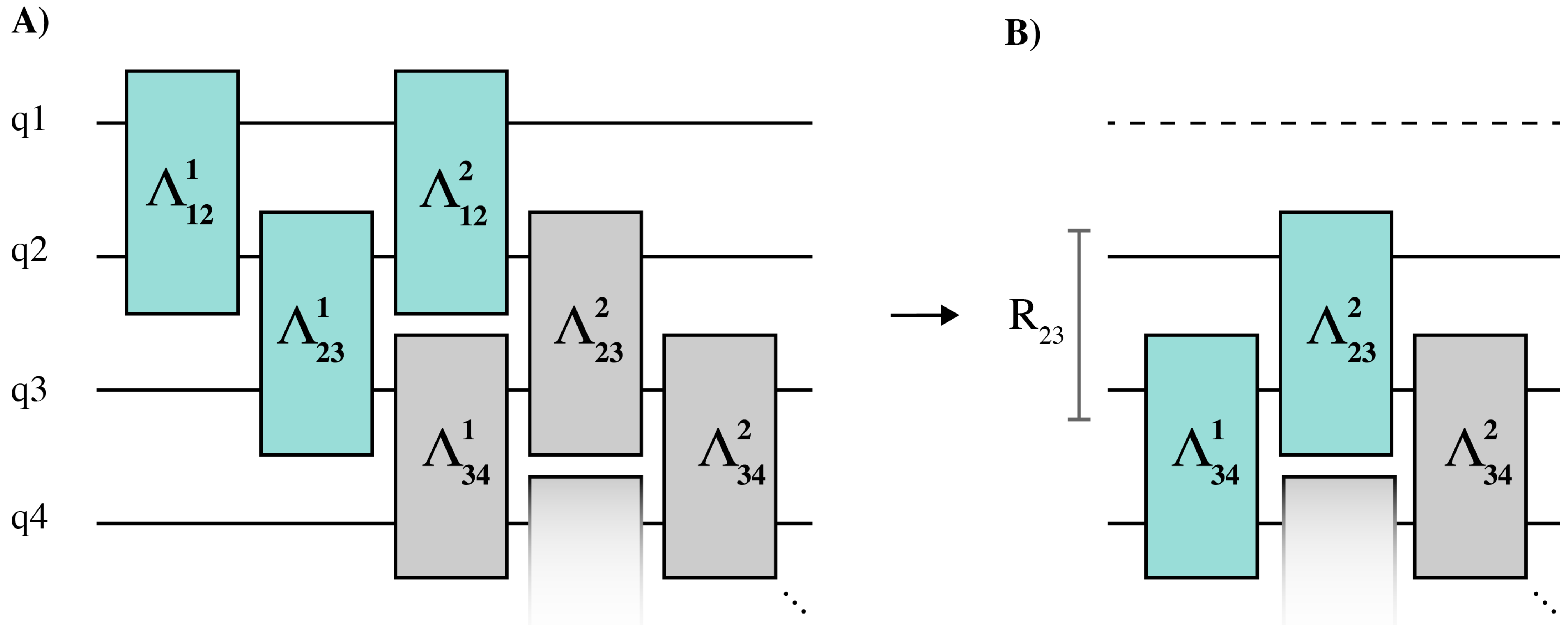

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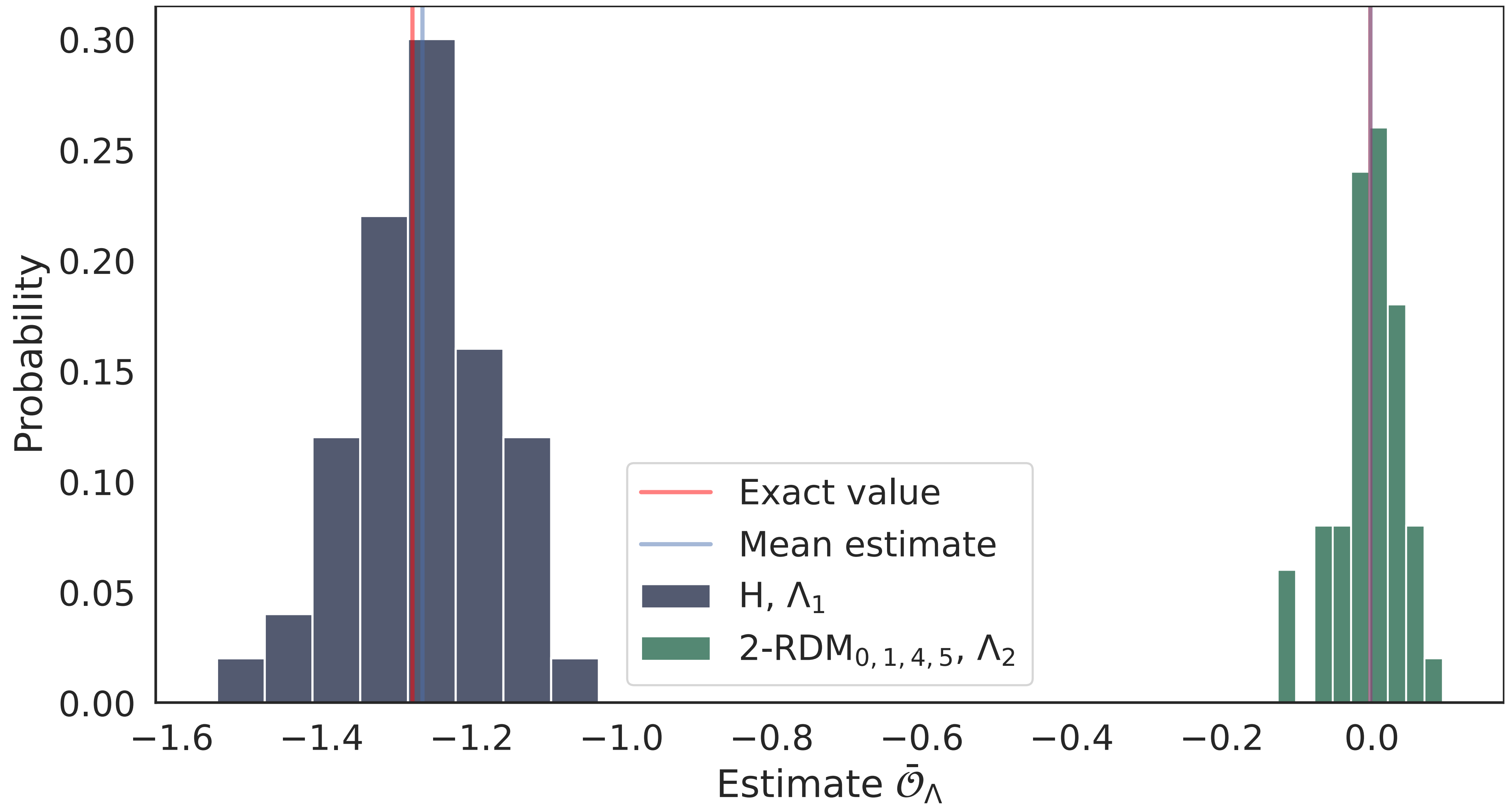
Traces involving mapped dual effects

- Computing the traces $\mathrm{Tr}[\Lambda(D_{\mathbf{m}_i})P_{\mathbf{k}}]$ in $\omega_{\mathbf{m}_i}$ can be challenging given the dimension of $\Lambda(D_{\mathbf{m}_i})$
- We exploit the causal cone structure of the circuit to bypass explicit high-dimensional reconstructions
- Algorithm polynomial in system size for many circuit topologies
- Can be executed *backwards* by exchanging the roles of $D_{\mathbf{m}_i}$ and $P_{\mathbf{k}}$, and using $\Lambda^\dagger$, the adjoint of the map

The VILMA method

operator averages on transformed states



- ρ is the ground state of H_2 perturbed by a CPTP map $\mathcal{N}$
- $S = 10^4$ and we repeat the experiment 50 times
- $\Lambda_0 = \mathbb{I} \,|\, \Lambda_1 = \mathcal{N}^{-1} \,|\, \Lambda_2$ is a layer of randomly chosen unitaries
- $\mathcal{O}_1 = H \,|\, \mathcal{O}_2 = \mathrm{Re}\langle a_2^\dagger a_7 \rangle \,|\, \mathcal{O}_3 = \mathrm{Re}\langle a_0^\dagger a_1 a_4^\dagger a_5 \rangle$

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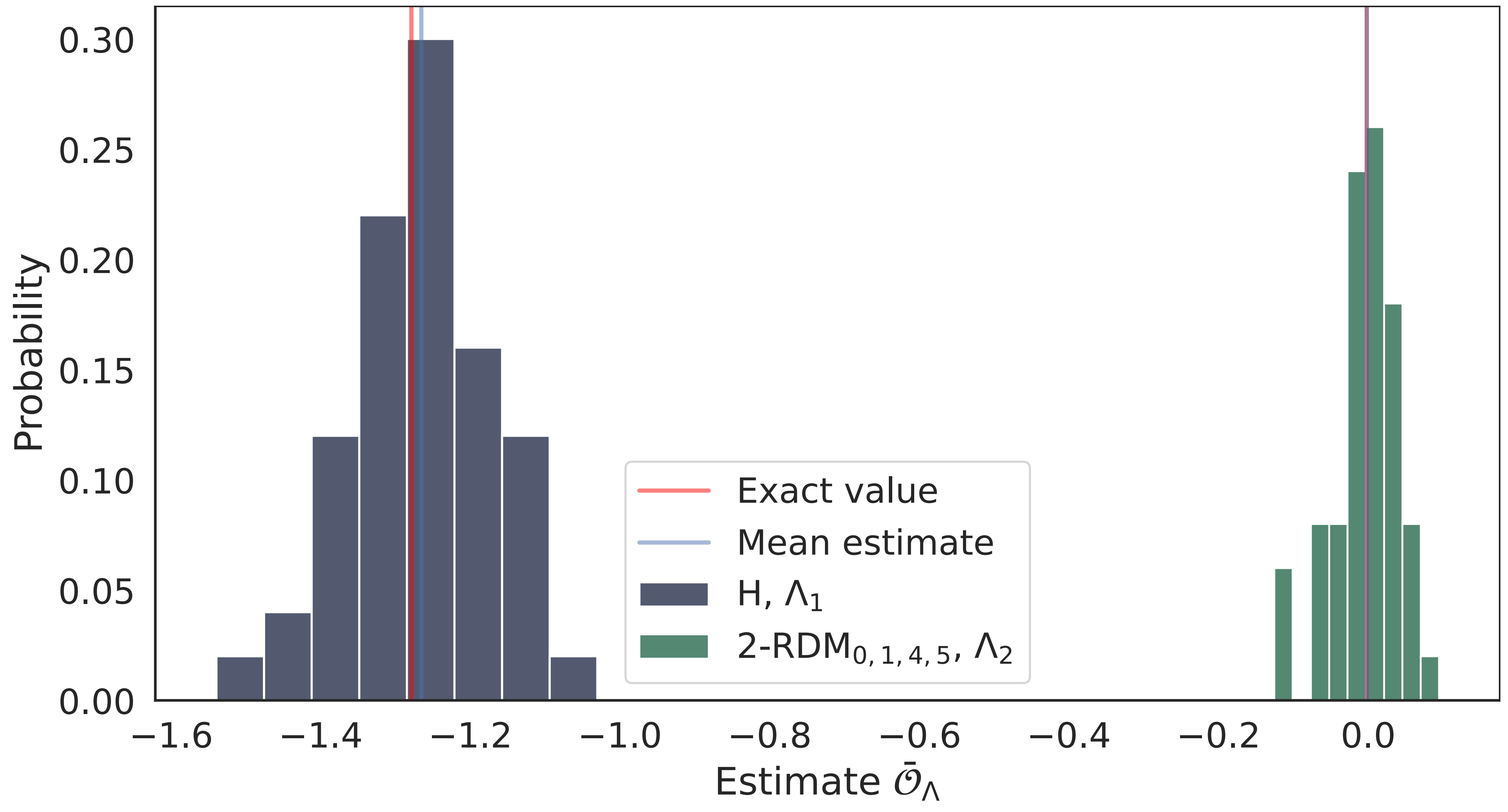

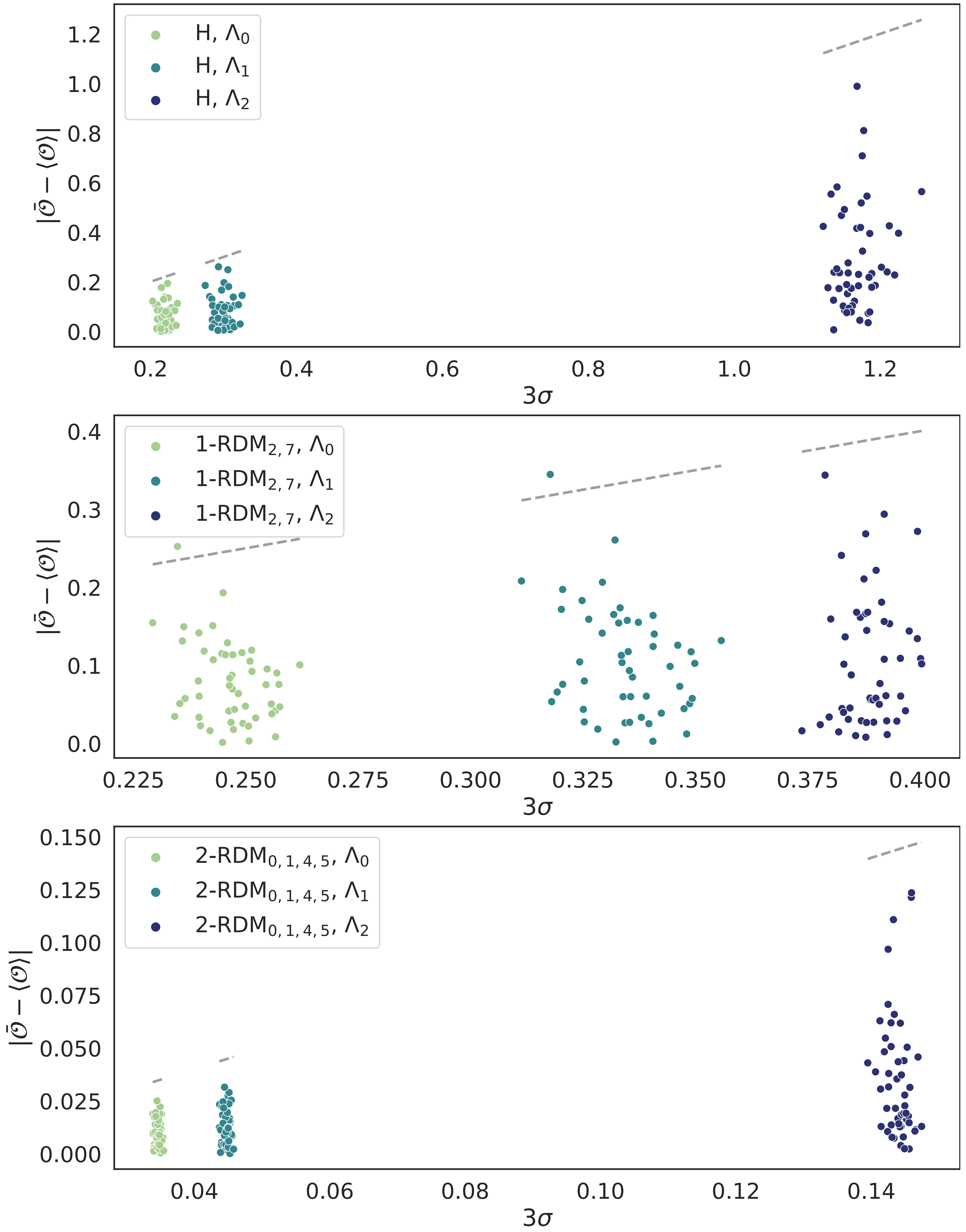

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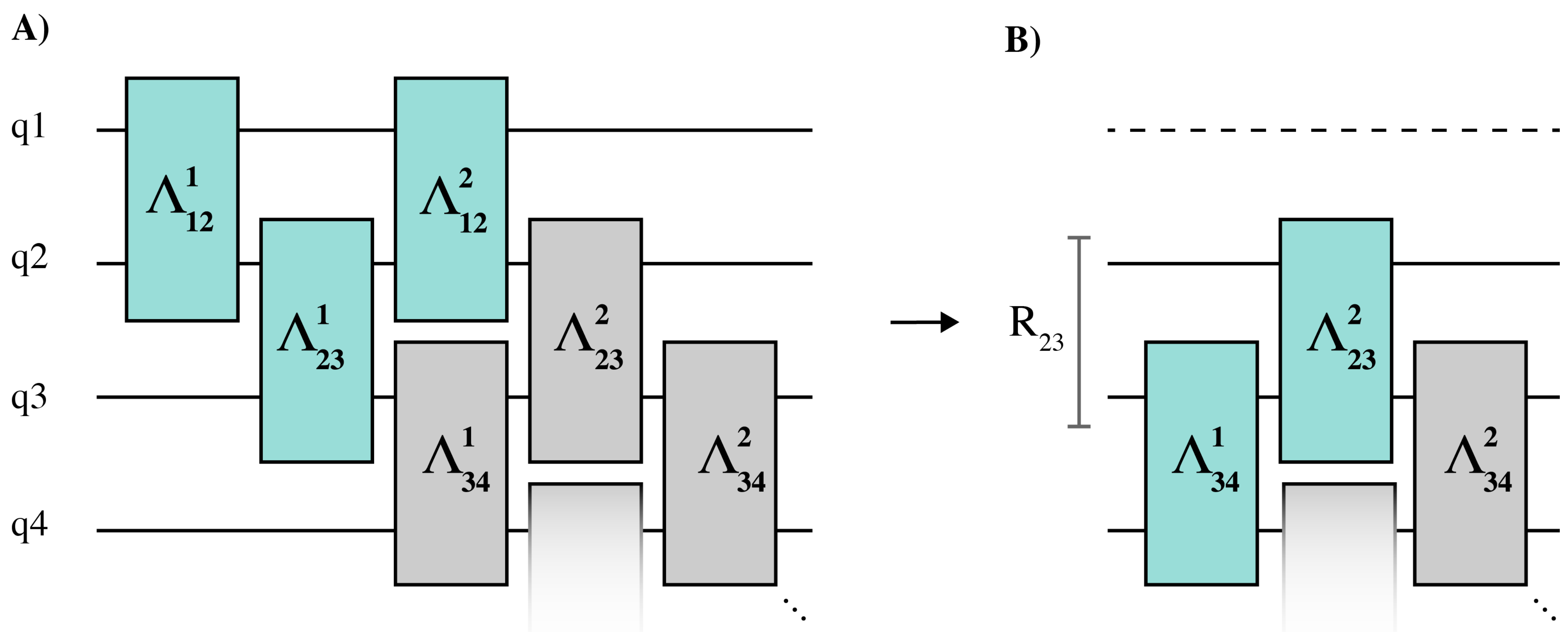
Variational optimisation with VILMA

observable dependence on local maps



Traces algorithm revisited

- By combining the forward and backwards algorithms, we can write an expression, linear in a singled-out map Λ^l_{ij}, that captures all the dependence of the observable average on Λ^l_{ij}

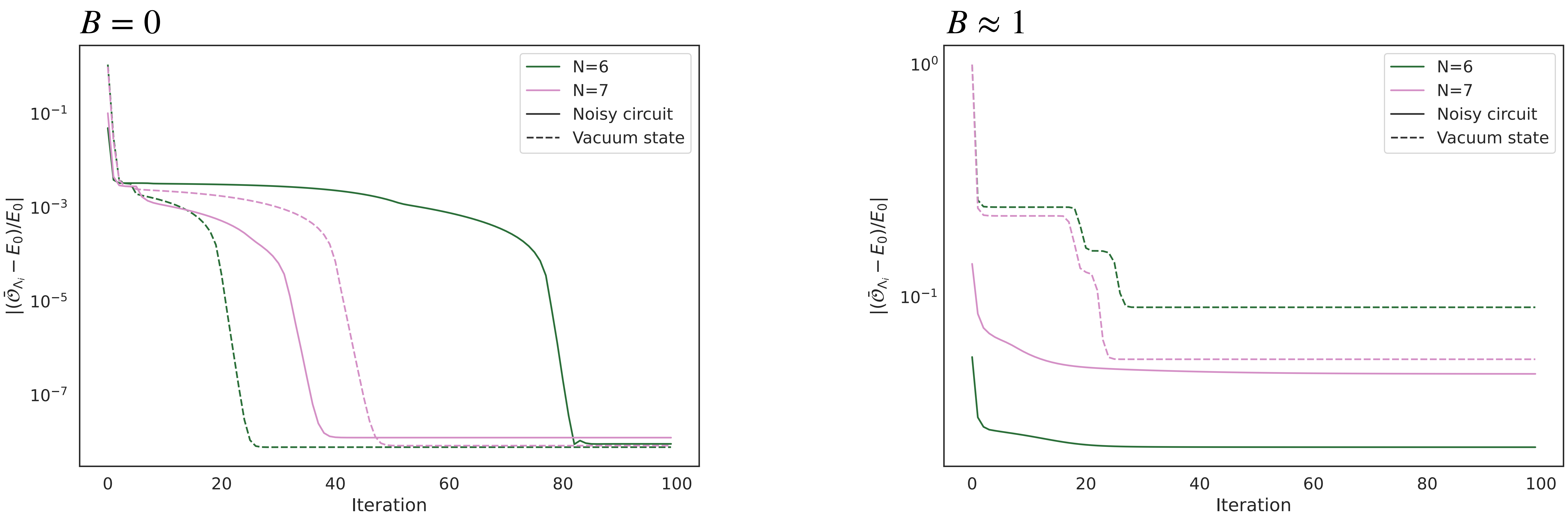
$$\bar{\mathcal{O}}_\Lambda = \sum_{i=1}^{S} \frac{1}{S} \sum_{\mathbf{k}} c_{\mathbf{k}} \sum_{a} \mathrm{Tr}[\Lambda^l_{ij}(R_a^{(\mathbf{m}_i,\mathbf{k})})\bar{R}_a^{(\mathbf{m}_i,\mathbf{k})}]$$

- The expression can be minimised/maximised efficiently while imposing *e.g.* positivity constraints using semi-definite programming

- In this context, it is interesting to explore "classical VILMA", in which the input data is simply $|0\rangle^{\otimes N}$

Variational optimisation with VILMA

observable dependence on local maps



- Solving for the ground state of the XX model $H = -J[\sum_i(\sigma_x^{(i)}\sigma_x^{(i+1)} + \sigma_y^{(i)}\sigma_y^{(i+1)})/2 + B\sigma_z^{(i)}]$ for $B = 0$ and $B \approx 1$
- Input states are a VQE circuit with noise (coherent and depolarising) in each CNOT and vacuum ($|0\rangle^{\otimes N}$)