Coherence Generation , Irreversible Entropy production and non-Adiabaticity in Quantum Processes

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Nemesys

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Outline:

1. Work production and irreversibility: Irreversible work

   1.1 Irreversibility due to Coherence generation

2. Non-Adiabaticity and irreversibility: Inner friction

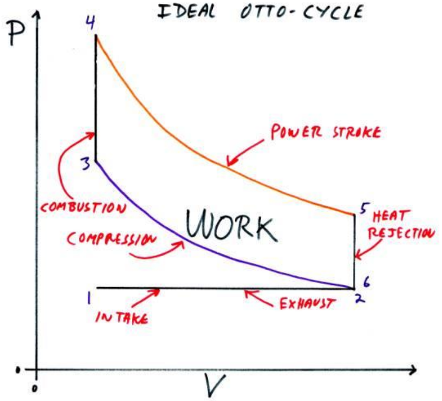
   2.1 Non-Adiabatic generation of quantum Coherence

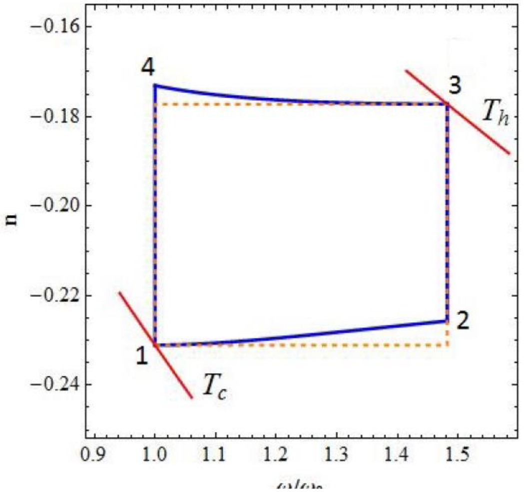
3. Conclusions:

   The role of coherence in Quantum Thermodynamics

Motivating example: single qubit Otto engine

$$H(\lambda(t)) = \frac{\omega_0}{2}\sigma_z + \lambda(t)\ \sigma_x \qquad \lambda(t) = \frac{\alpha\omega_0 t}{2}$$





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PAPER

Quantum Otto cycle with inner friction: finite-time and disorder effects

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Work production and Irreversibility

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Irreversible Work and Inner Friction in Quantum Thermodynamic Processes

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PHYSICAL REVIEW E **99**, 042105 (2019)

Role of coherence in the nonequilibrium thermodynamics of quantum systems

G. Francica,[1,2] J. Goold,[3] and F. Plastina[1,2]

Setting the stage: Thermodynamic transformation

Work parameter $\lambda(t)$ **:** $\lambda(t=0)=\lambda_i \longrightarrow \lambda(\tau)=\lambda_f$

Closed quantum system :

the Hamiltonian $H[\lambda(t)]$ generates the evolution $U(\tau,0)$

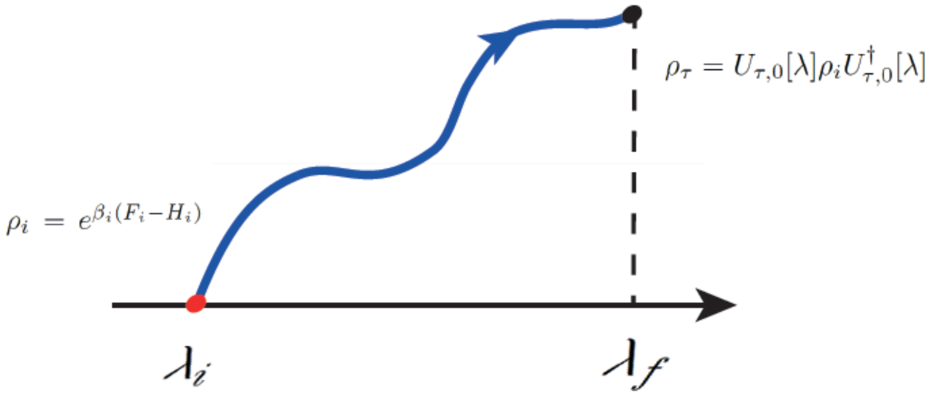
Initial (equilibrium) state

$$\rho_i = e^{-\beta_i H_i}/Z[\lambda_i,\beta_i]$$

↓

final state

$$\rho_\tau = U(\tau,0)\rho_i U^\dagger(\tau,0)$$



Work and Jarzynski relation

Probability density for the work done on the system:

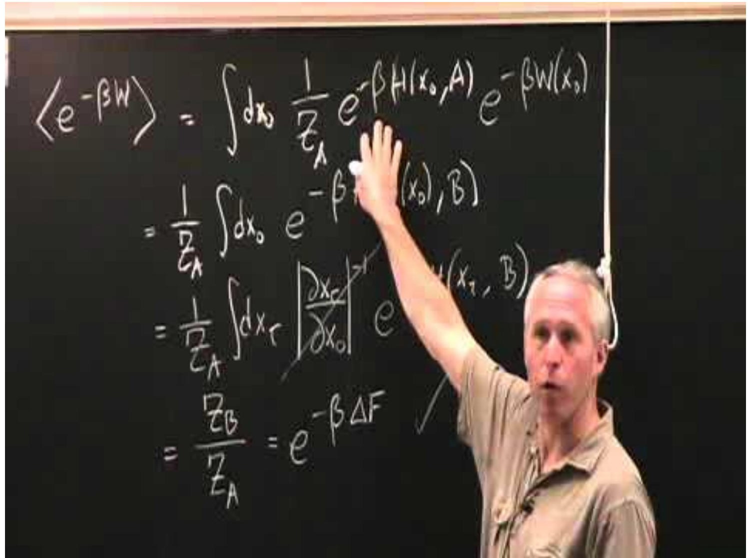
$$p(w) = \sum_{n,m} P_n^{(i)} \, P_{n \to m}^{(\tau)} \, \delta(w - \varepsilon_m^{(f)} + \varepsilon_n^{(i)})$$

with $\quad P_n^{(i)} = Z_i^{-1} e^{-\beta_i \varepsilon_n^{(i)}} \quad$ and $\quad P_{n \to m}^{(\tau)} = \left| \left\langle \varepsilon_m^{(f)} \right| U(\tau, 0) \left| \varepsilon_n^{(i)} \right\rangle \right|^2$

Fluctuation relation

$$\left\langle e^{-\beta_i w} \right\rangle = e^{-\beta_i \Delta F}$$

where $\quad \Delta F = F[\lambda_f, \beta_B] - F[\lambda_i, \beta_i]$



Irreversible Work

$$\langle e^{-\beta_i w} \rangle = e^{-\beta_i \Delta F} \quad \Longrightarrow \quad \langle w_{irr} \rangle = \langle w \rangle - \Delta F \geq 0$$

The irreversible work quantifies the irreversibility

Irreversible Work

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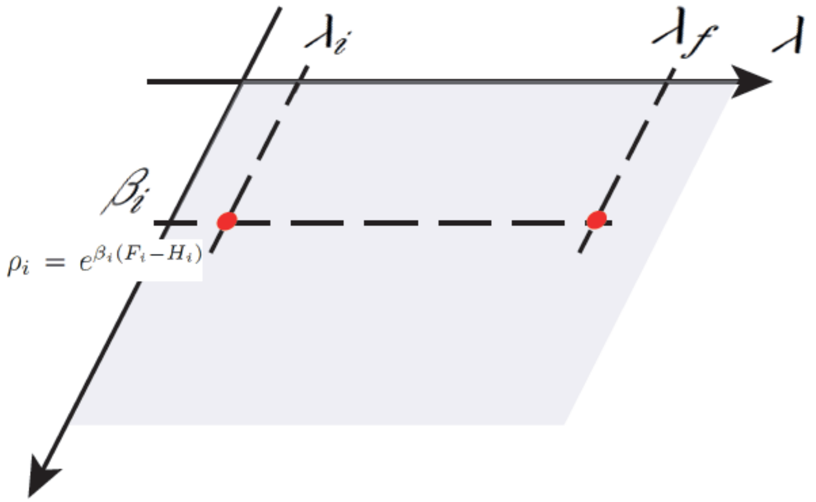

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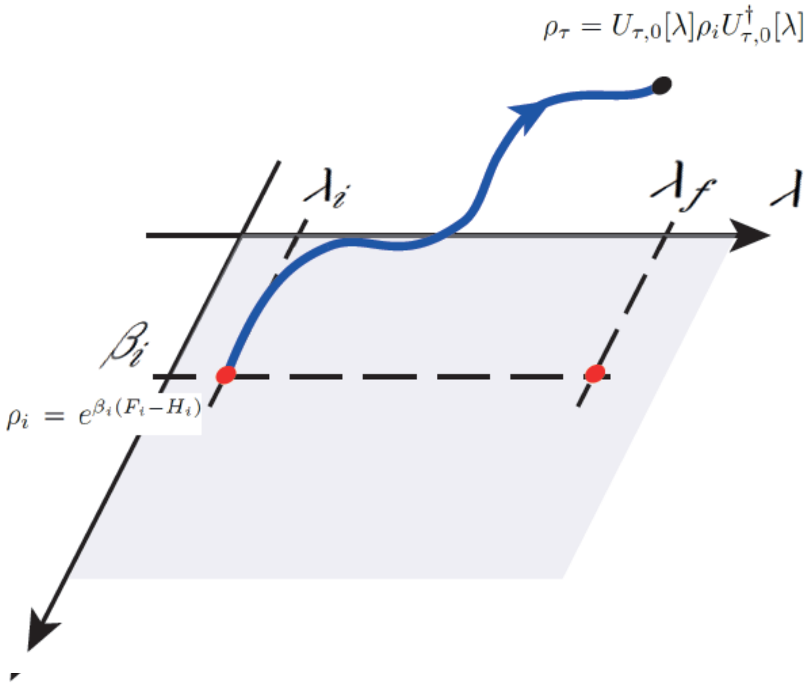

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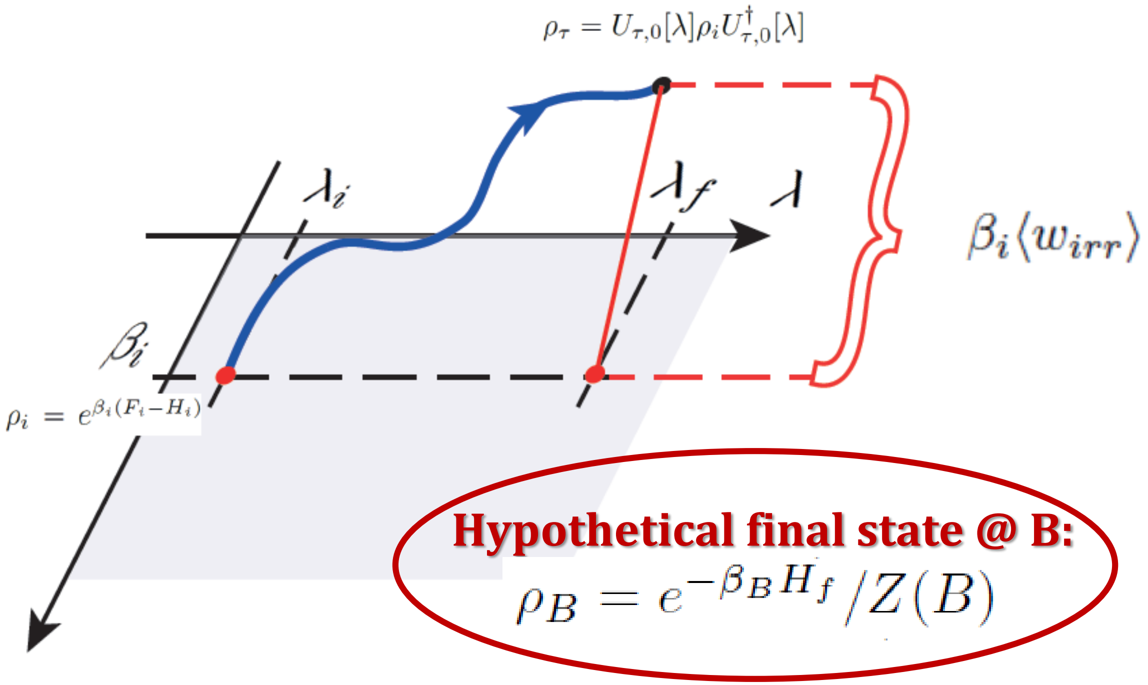
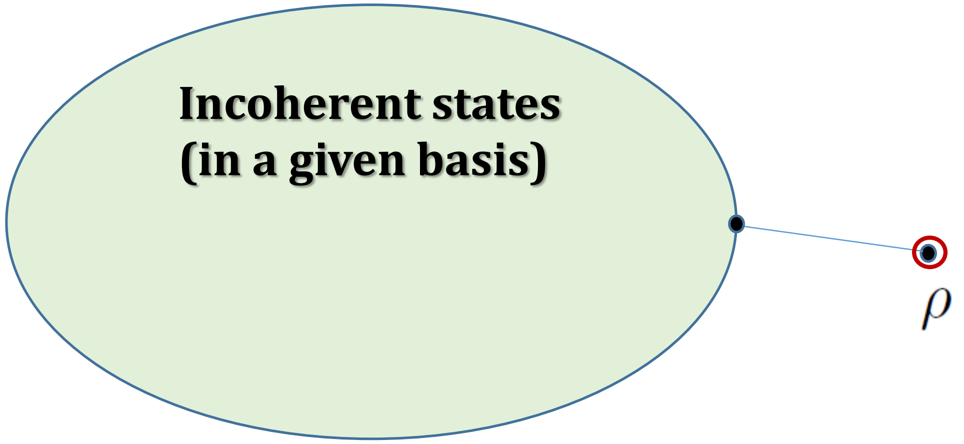


quantum relative entropy

$$\langle S_{irr} \rangle = \beta_i \langle w_{irr} \rangle = D(\rho_\tau || \rho_B)$$

S. Deffner and E. Lutz, PRL10

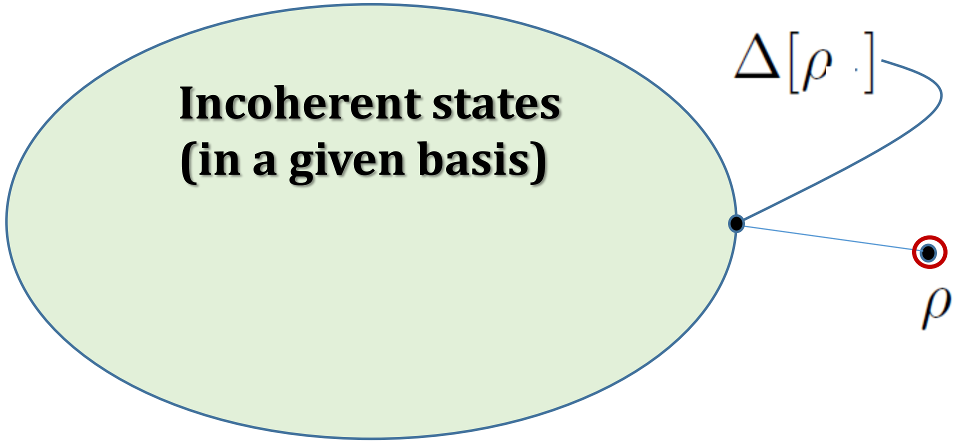
(entropic) Quantification of Coherence



definition

$$C(\rho) = \min_{\sigma \in I} D(\rho || \sigma)$$

(entropic) Quantification of Coherence



definition

$$C(\rho) = \min_{\sigma \in I} D(\rho || \sigma)$$

minimization

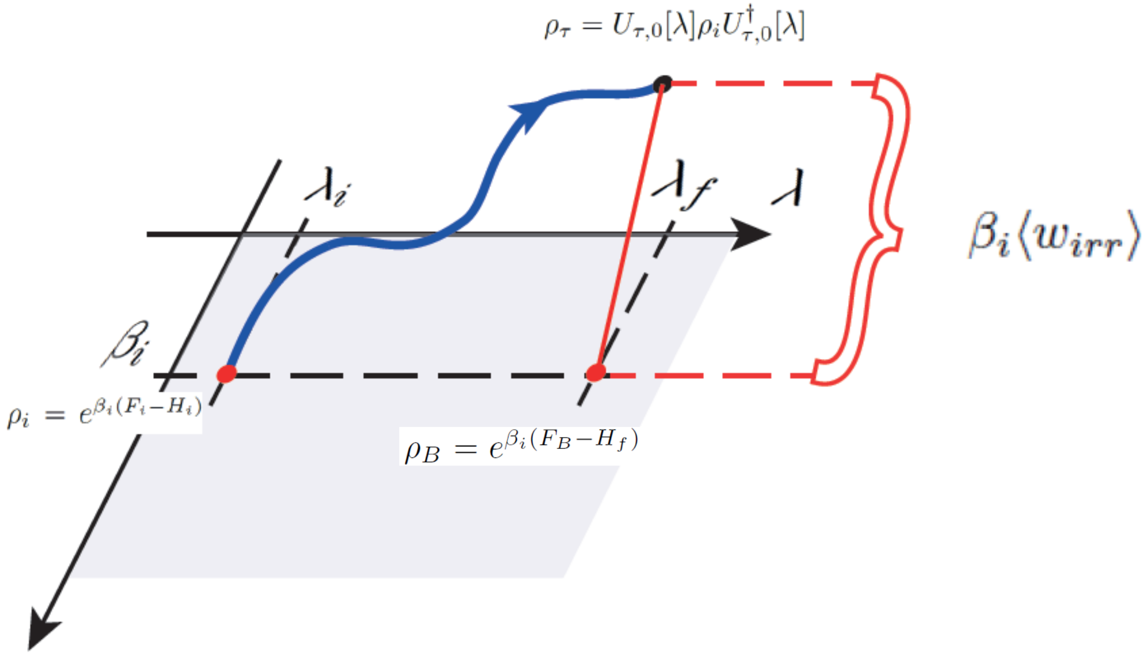
$$C(\rho) = D(\rho || \Delta[\rho]) = S(\Delta[\rho]) - S(\rho)$$

A. Streltsov, G. Adesso, M. Plenio, ArXiv 2016

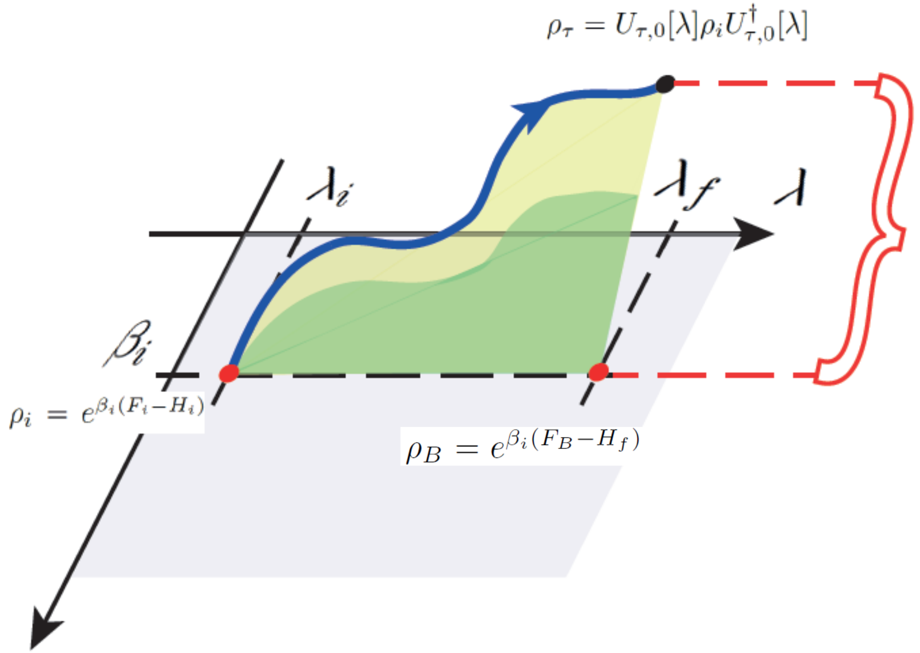
In the thermodynamic context, the "preferred" basis is obvious !

Generated Coherence : $C(\rho_\tau) = D(\rho_\tau || \Delta[\rho_\tau]) = S(\Delta[\rho_\tau]) - S(\rho_i)$

Irreversible work and coherence generation



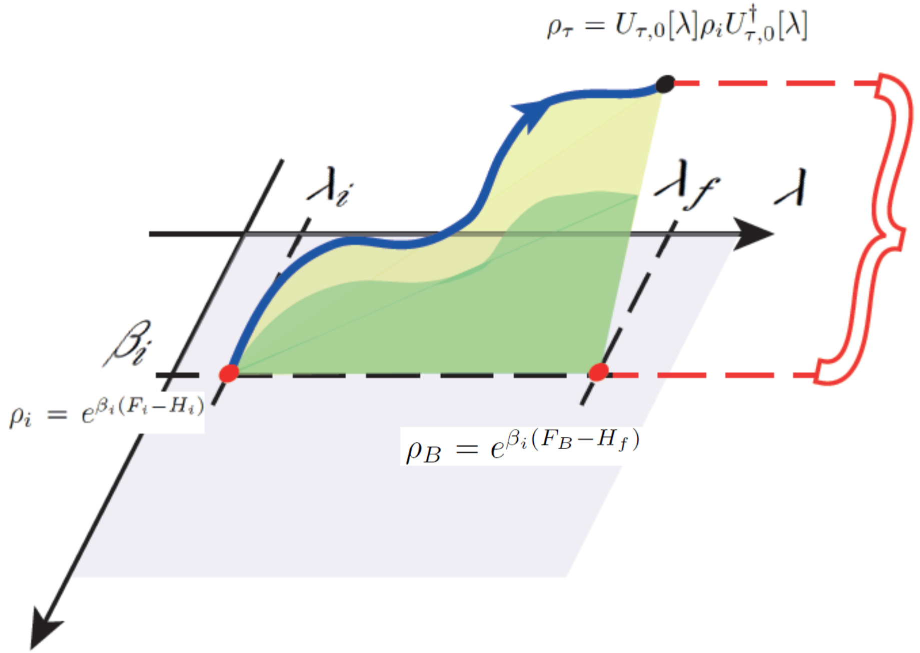
Irreversible work and coherence generation



$$\beta_i \langle w_{irr} \rangle = C(\rho_\tau) + D(\Delta[\rho_\tau] || \rho_B)$$

In fact, the decomposition holds at any intermediate time t.

Irreversible work and coherence generation



$$\beta_i\langle w_{irr}\rangle = C(\rho_\tau) + D(\Delta[\rho_\tau]||\rho_B)$$

In fact, the decomposition holds at any intermediate time t.

Three fluctuation theorems:

Stochastic variables

$$s_{nm} := \beta_i[(\epsilon_m(\tau) - \epsilon_n(0)) - (F_B - F_i)]\,,$$
$$p_{nm} := \ln\rho_{mm}(\tau) - \ln\rho_{B,mm}\,,$$
$$c_{nm} := s_{nm} - p_{nm} = \ln\rho_{nn}(0) - \ln\rho_{mm}(\tau)$$

$$P(\alpha) = \sum_{n,m}\rho_{nn}(0)P_{n\to m}(\tau)\,\delta(\alpha - \alpha_{nm})\,, \quad \text{for } \alpha = s, p, c\,,$$

averages

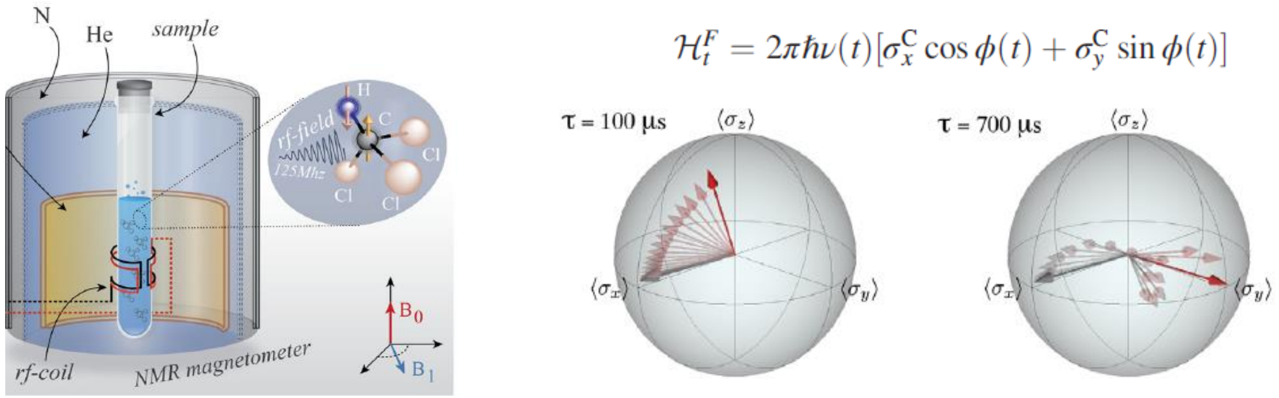
$$\langle s\rangle \equiv \langle S_{irr}\rangle\,,\ \langle p\rangle \equiv D(\Delta_\tau[\rho_\tau]||\rho_B)\,,\ \langle c\rangle \equiv C(\rho_\tau)$$

$$\langle s\rangle = \langle p\rangle + \langle c\rangle$$

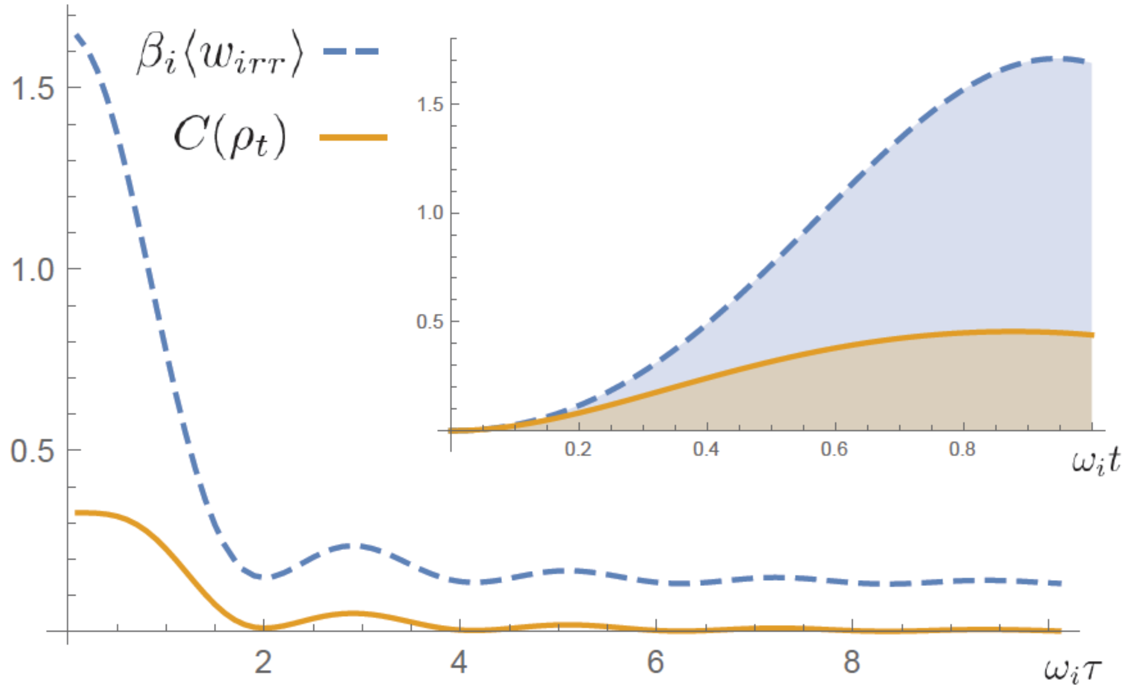
Fluctuation relations

$$\langle e^{-s}\rangle = \langle e^{-c}\rangle = \langle e^{-p}\rangle = 1$$

Example 1: spin ½ in a rotating magnetic field



Experiment:
T. B. Batalhao et al., PRL 2015



Example 2: kicked rotor

model

$$H(t) = H_0 + V(\theta)\sum_{n\in\mathbb{Z}}\delta(t-nT)$$

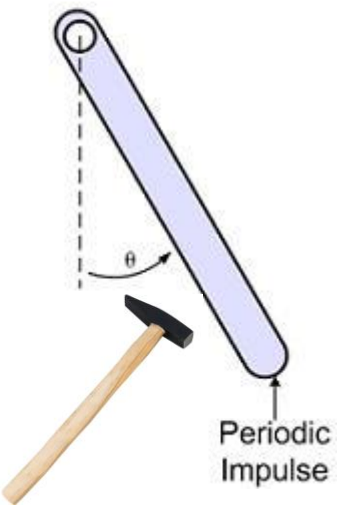
$$\hat{H}_0 = \frac{\hat{p}^2}{2} \quad \text{with } \hat{p} = -i\partial_\theta, \text{ and } \theta \in [0, 2\pi)$$

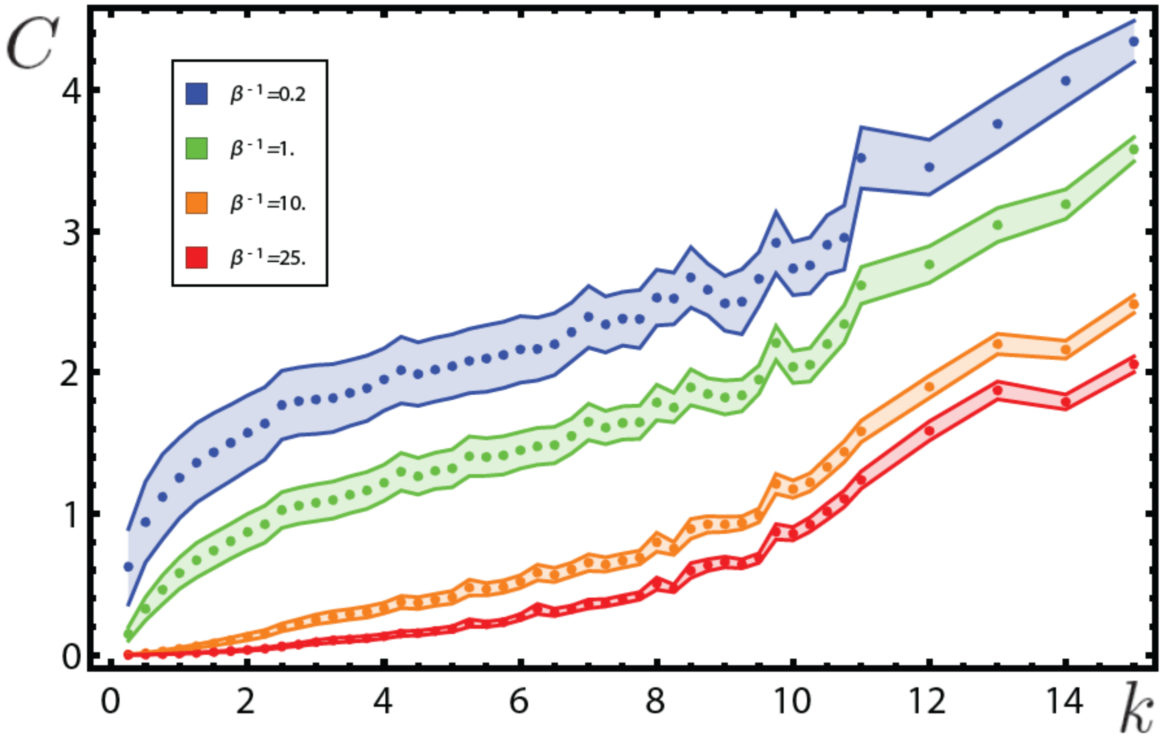
$$V(\theta) = k\cos(\theta)$$

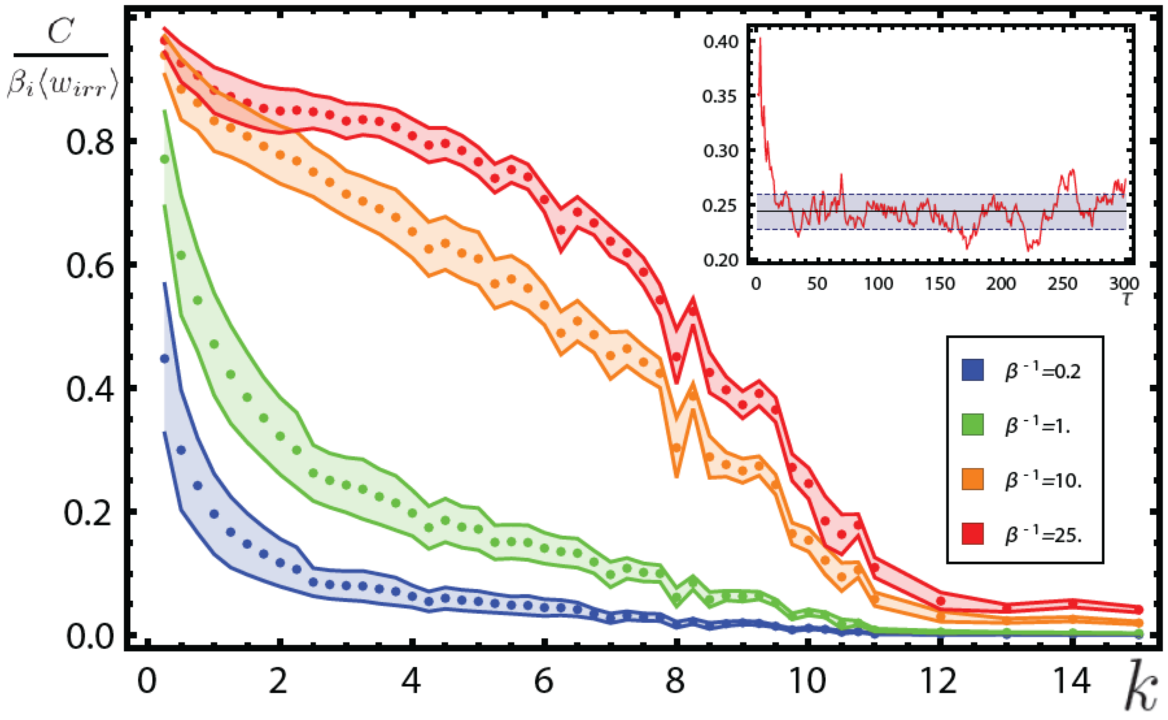
evolution

$$\hat{U} = e^{-i\hat{H}_0 T} e^{-iV(\hat{\theta})}$$

$$\rho_\tau = U^\tau \rho_0 U^{\tau\dagger}$$





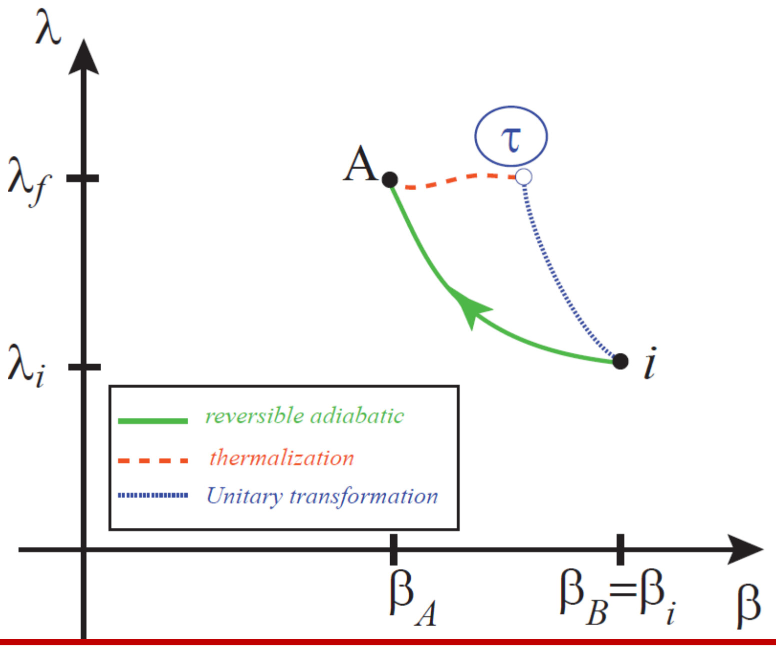


Adiabatic Transformation and non-adiabaticity

Define the adiabatic state A by the relation: $\rho_A \;:=\; U_A \rho_0 U_A^\dagger$ **(*)** $\qquad U_A \;=\; \lim_{\tau\to\infty} U_{\tau,0}[\lambda]$

(entropic) non-adiabaticity parameter:

$$\mathcal{A} \;:=\; D(\rho_\tau || \rho_A)$$



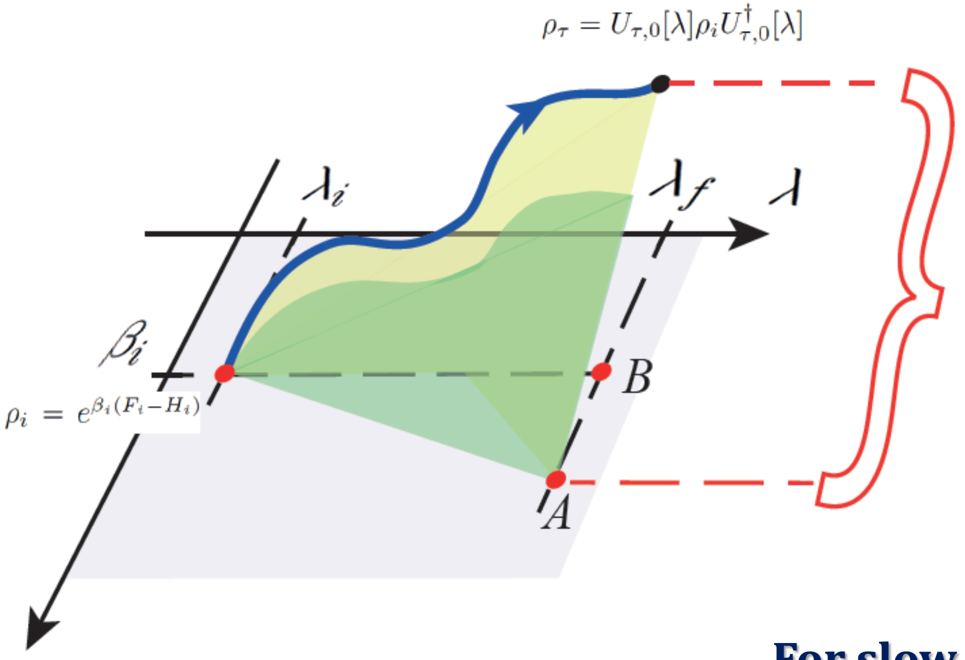
Adiabatic Work (hypothetical final state @ A)

$$\langle w_{i\to A}\rangle = \mathcal{U}_A - \mathcal{U}_i \equiv \sum_m P_m^{(i)}(\varepsilon_m^{(f)} - \varepsilon_m^{(i)})$$

Inner friction

$$\langle w_{fric}\rangle = \langle w\rangle - \langle w_{i\to A}\rangle = \frac{1}{\beta_A} D(\rho_\tau || \rho_A)$$

Non-adiabaticity and coherence generation



$$\mathcal{A} = C(\rho_\tau) + D(\Delta_\tau[\rho_\tau]||\rho_A)$$

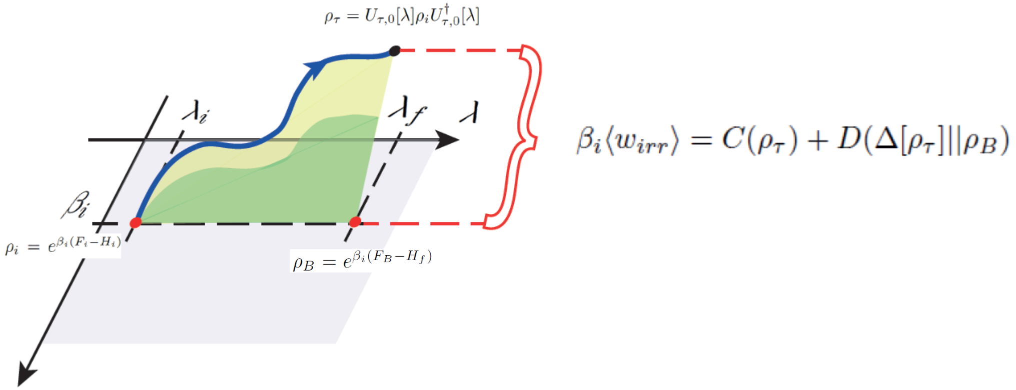
For slow (but yet non-adiabatic) processes,
Coherence generation is the dominant contribution to non-adiabaticity :

$$\mathcal{A} = C(\rho_\tau) + \mathcal{O}\left(\frac{1}{\tau^2}\right)$$

Summary:

1. Non-equilibrium thermodynamics

irreversibility due to coherence generation



2. Non-adiabaticity

inner friction