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Large deviation theory of quantum jump statistics in a chiral waveguide

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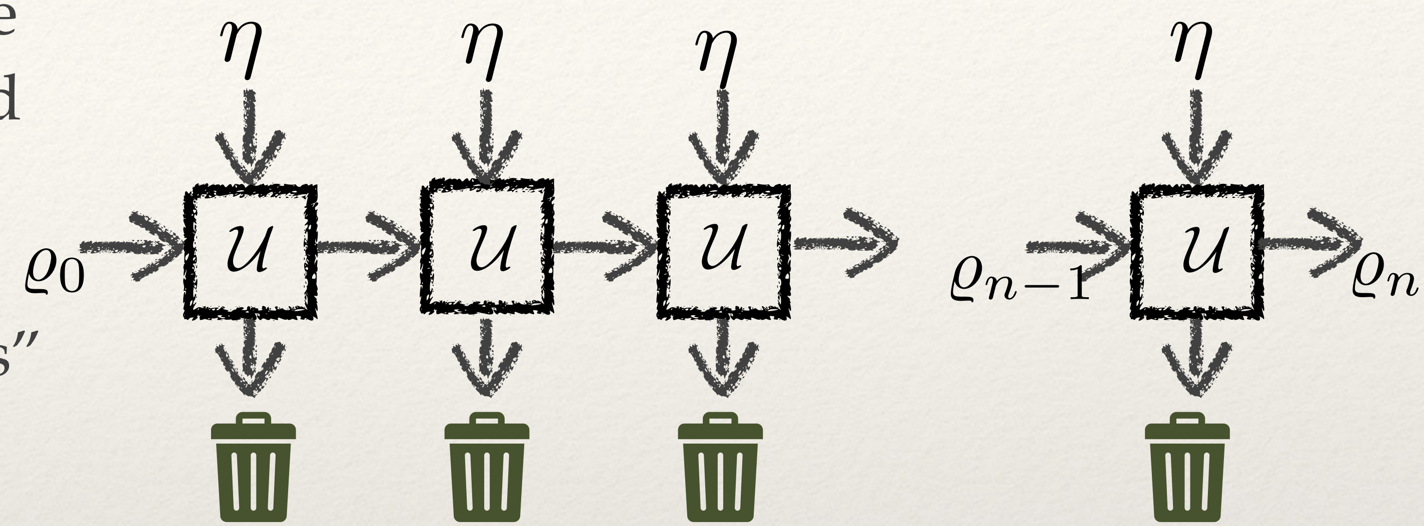
collision models of open quantum systems dynamics

- the environment consists of a very large number of identical subsistems, dubbed “ancillas”, initially all in the same state
- the system environment interaction takes place via a sequence of “collisions” between the system and the ancillas i.e. via pairwise short unitary interactions lasting a short time
- equivalent to standard Born-Markov approximation
- the collision time τ plays the role of the coherence time for the environment



$$\mathcal{U} = e^{-iV\tau/\hbar} \simeq \mathcal{I} - \frac{-iV\tau}{\hbar} + \frac{(V\tau)^2}{2\hbar^2}$$

$$\dot{\rho} = -i\,[\hat{H}, \rho] + \mathcal{D}(\hat{L})\,\rho\,,$$

$$\eta = |e_0\rangle\langle e_0|$$

$$\mathcal{D}(\hat{L})\,\rho = \hat{L}\,\rho\,\hat{L}^\dagger - \tfrac{1}{2}\,(\hat{L}^\dagger\hat{L}\rho + \rho\,\hat{L}^\dagger\hat{L})\,,$$

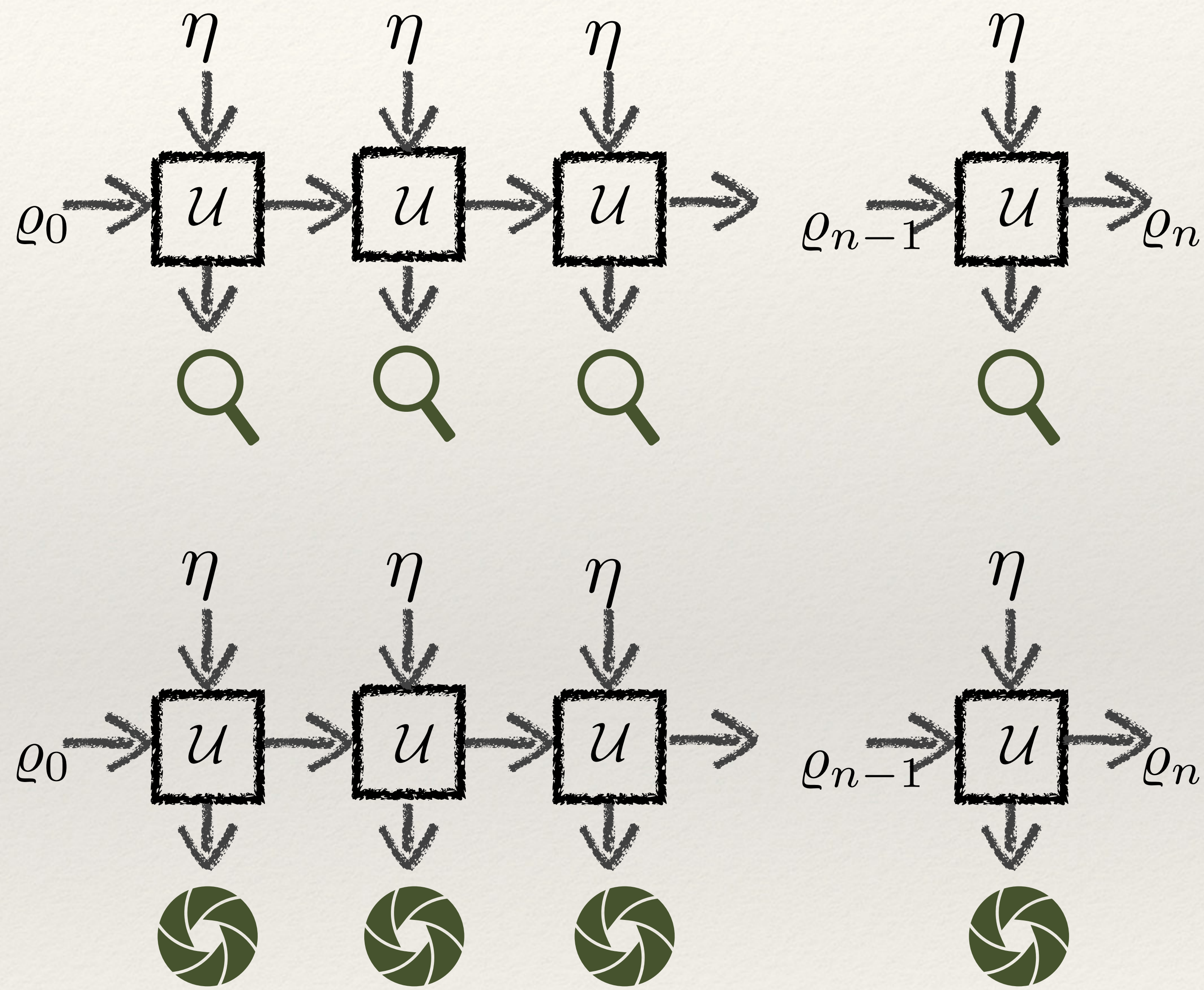
$$\hat{L}_k = \langle e_k|\hat{V}|e_0\rangle\sqrt{\tau/\hbar}$$

jump operator

master equation in Lindbad form

the unraveling

- tracing over the environment leads to a master equation giving information on the average behaviour of the system
- if the environment is measured at each step the system undergoes a series of jumps
- the master equation is recovered as an average over the unraveling, i.e. as an average ove different histories
- different detections can give rise to different unraveling describing the same master equation



$$|\psi\rangle|e_0\rangle \to |\psi\rangle|e_0\rangle - \sqrt{\tau}\sum_k \hat{L}_k|\psi\rangle|e_k\rangle - (\tau/2)\sum_k \hat{L}_k\hat{L}_k|\psi\rangle|e_0\rangle$$

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Thermodynamics of Quantum Jump Trajectories

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$$P_t(K) = \mathrm{Tr}[\rho^{(K)}(t)] \approx e^{-t\varphi(K/t)}. \tag{2}$$

The "large-deviation" function $\varphi(k)$ ($k \equiv K/t$) contains all information about the probability of K at long times [2]. Alternatively, we can describe the statistics of K via the generating function, which also has a LD form [2],

$$Z_t(s) \equiv \sum_{K=0}^{\infty} P_t(K) e^{-sK} \approx e^{t\theta(s)}. \tag{3}$$

Derivatives of $\theta(s)$ give moments of the photon number distribution. In particular, the average number of emitted photons is $k_0 \equiv \langle K \rangle / t = -\theta'(0)$, and the Mandel parameter, $Q_0 \equiv (\langle K^2 \rangle - \langle K \rangle^2)/\langle K \rangle - 1 = -\theta''(0)/\theta'(0)$. The LD function around $s = 0$ encodes the information about fluctuations of *typical* trajectories [4,10].

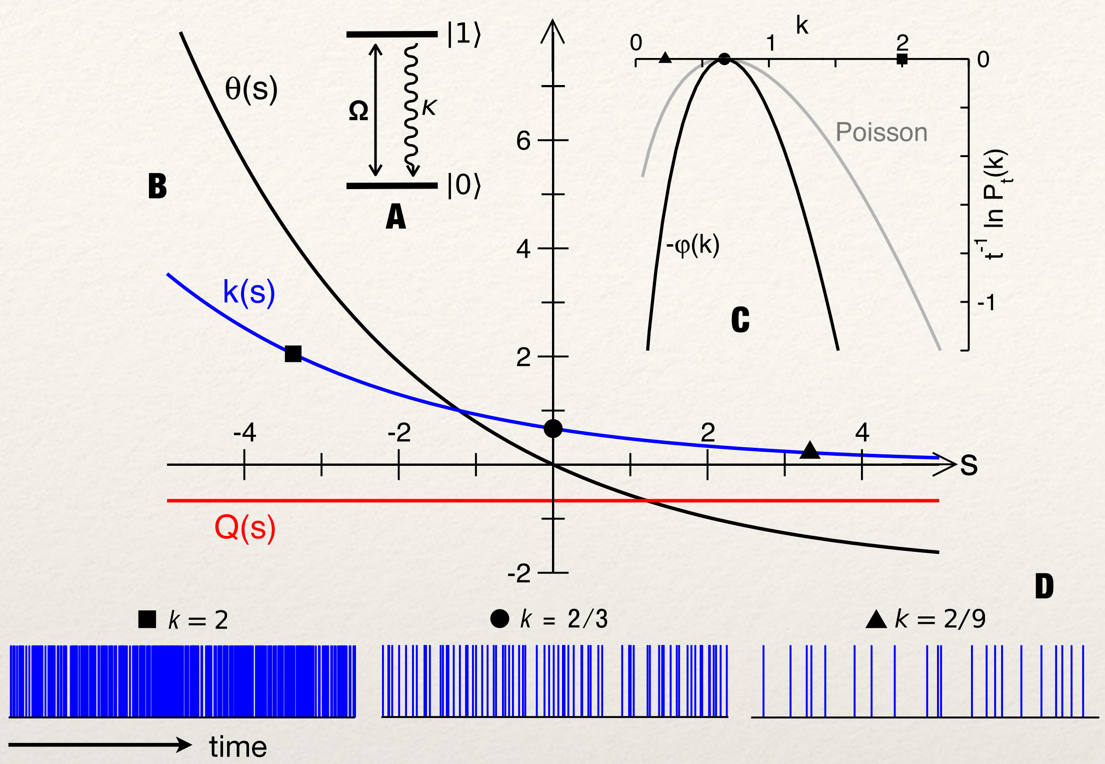

FIG. 1 (color online). (A) Laser driven 2-level system coupled to a $T = 0$ bath. (B) Large-deviation function $\theta(s)$. Dynamical trajectories go from more active to less active as s, the conjugate field to the number of emitted photons K, is increased, as shown by the average photon rate $k(s) \equiv \langle K \rangle_s / t = -\theta'(s)$. The Mandel parameter $Q(s) = -2/3$ for all s, indicating that for $\kappa = 4\Omega$ trajectories display a form of scale invariance. (C) The photon count probability is obtained from (5) by a Legendre transform: $P_t(K) \approx e^{-t\varphi(K/t)}$ with $\varphi(k) = 3[k \ln(k/k_0) - (k - k_0)]$. It is a $\nu = 3$ Conway-Maxwell-Poisson distribution [20], $P_t(K) \propto [\mathrm{Poisson}(K;t)]^3$. (D) Representative trajectories from subensembles with different average k.

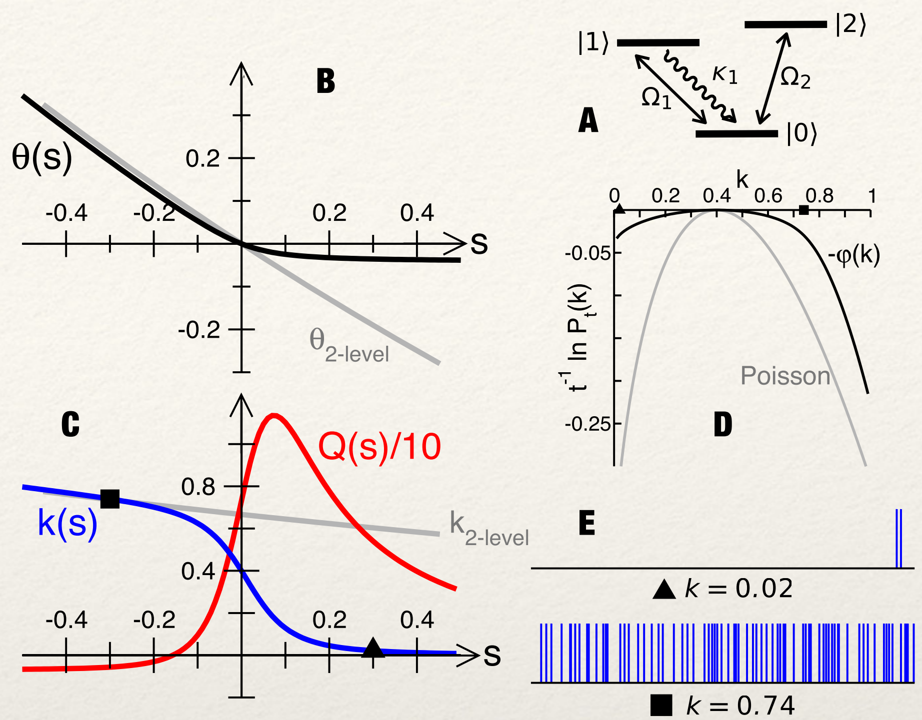

FIG. 2 (color online). (A) Laser driven 3-level system. Here $\kappa_1 = 4\Omega_1$ and $\Omega_2 = \Omega_1/10$. (B, C) The LD function $\theta(s)$ and dynamical order parameter $k(s)$ display crossover behavior near $s = 0$ between active and inactive dynamical regimes. The active side is antibunched, $Q < 0$. The inactive side is nonfluctuating $Q = 0$. The peak in Q near $s = 0$ signals the dynamical crossover. (D) The fat tail for $k < k_0$ in $P_t(K)$ is a manifestation of the inactive regime; the thin tail for $k > k_0$ is a manifestation of the active regime. (E) Representative trajectories from inactive and active subensembles. At $s = 0$ there is (mesoscopic) coexistence of the two dynamical regimes and typical trajectories are intermittent or "blinking."

Quantum jump statistics with a shifted jump operator in a chiral waveguide

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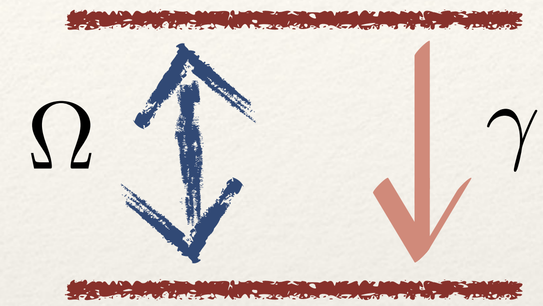
Resonance Fluorescence

$$\dot{\rho} = -i\,[\hat{H}, \rho] + \mathcal{D}(\hat{L})\,\rho\,,$$



$$\mathcal{D}(\hat{L})\,\rho = \hat{L}\,\rho\,\hat{L}^\dagger - \tfrac{1}{2}\,(\hat{L}^\dagger\hat{L}\rho + \rho\,\hat{L}^\dagger\hat{L})\,,$$

where the driving Hamiltonian $\hat{H}$ and jump operator $\hat{L}$ are respectively given by

$$\hat{H} = \Omega\,(\hat{\sigma}_+ + \hat{\sigma}_-)\;, \qquad\qquad \hat{L} = \sqrt{\gamma}\,\hat{\sigma}_-\,. \tag{3}$$

ME (1) can be "unraveled" in terms of (3) by expressing its solution at time t as

$$\rho_t = \mathcal{R}_t\rho_0 + \sum_{K=1}^{\infty}\int_0^t dt_K \cdots \int_0^{t_2} dt_1\,\mathcal{R}_{t-t_K}\mathcal{J}\mathcal{R}_{t_K-t_{K-1}}\cdots\mathcal{J}\mathcal{R}_{t_2-t_1}\mathcal{J}\mathcal{R}_{t_1}\rho_0\,, \tag{4}$$

where we defined the superoperators

$$\mathcal{R}_t\,\rho = e^{-i\left(\hat{H}-\frac{i}{2}\hat{L}^\dagger\hat{L}\right)t}\rho\,e^{i\left(\hat{H}-\frac{i}{2}\hat{L}^\dagger\hat{L}\right)t}, \quad \mathcal{J}\rho = \hat{L}\,\rho\,\hat{L}^\dagger\,. \tag{5}$$

A class of unravelings for the resonance fluorescence ME (3) is defined by

$$\hat{H} = \left(\Omega - \tfrac{1}{2}\sqrt{\gamma}\,\alpha\right)\hat{\sigma}_+ + \text{H.c.}\,, \quad \hat{L} = \sqrt{\gamma}\,\hat{\sigma}_- + i\alpha\ ,$$

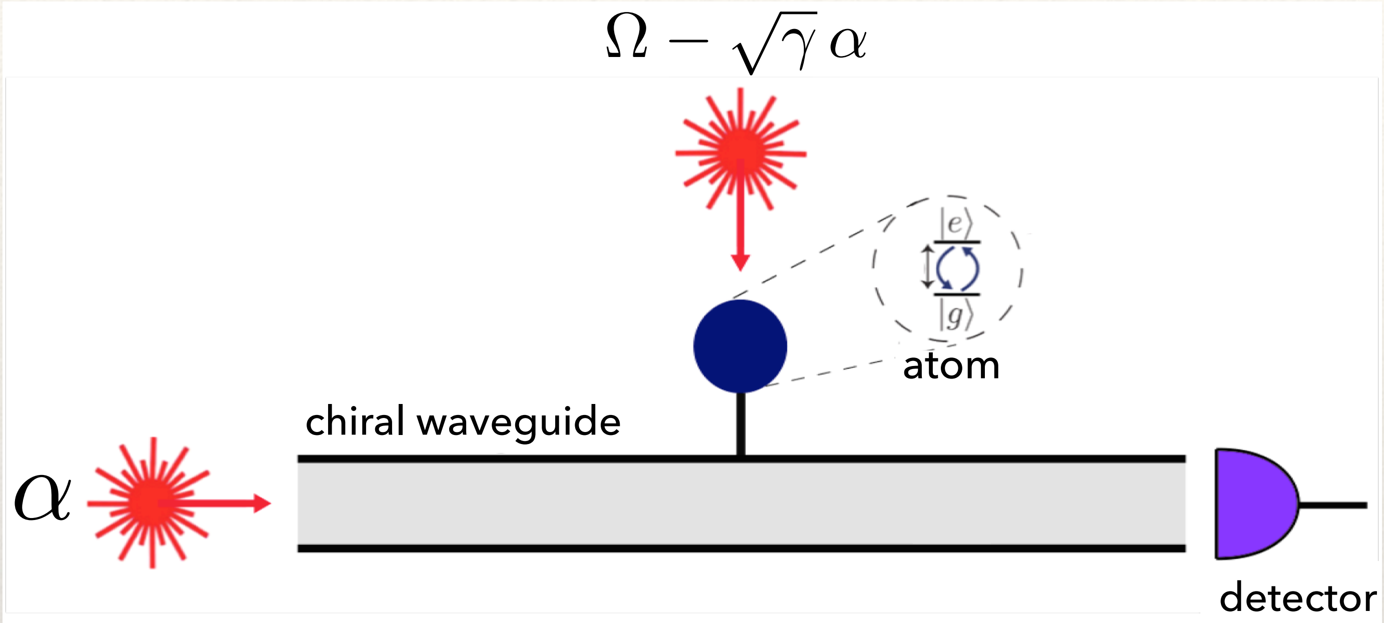


Figure 1. Waveguide-QED setup to implement unraveling (8) with shifted jump operator. A two-level atom is coupled to a chiral waveguide, sustaining a one-dimensional field that propagates in one direction only. The atom is driven by two continuous-wave coherent beams: one with Rabi frequency $\Omega - \sqrt{\gamma}\,\alpha$ propagating out of the waveguide and another one of amplitude α traveling along of the waveguide. Photons are counted at one end of the waveguide: the detector senses light emitted from the atom superposed with the coherent wave α.

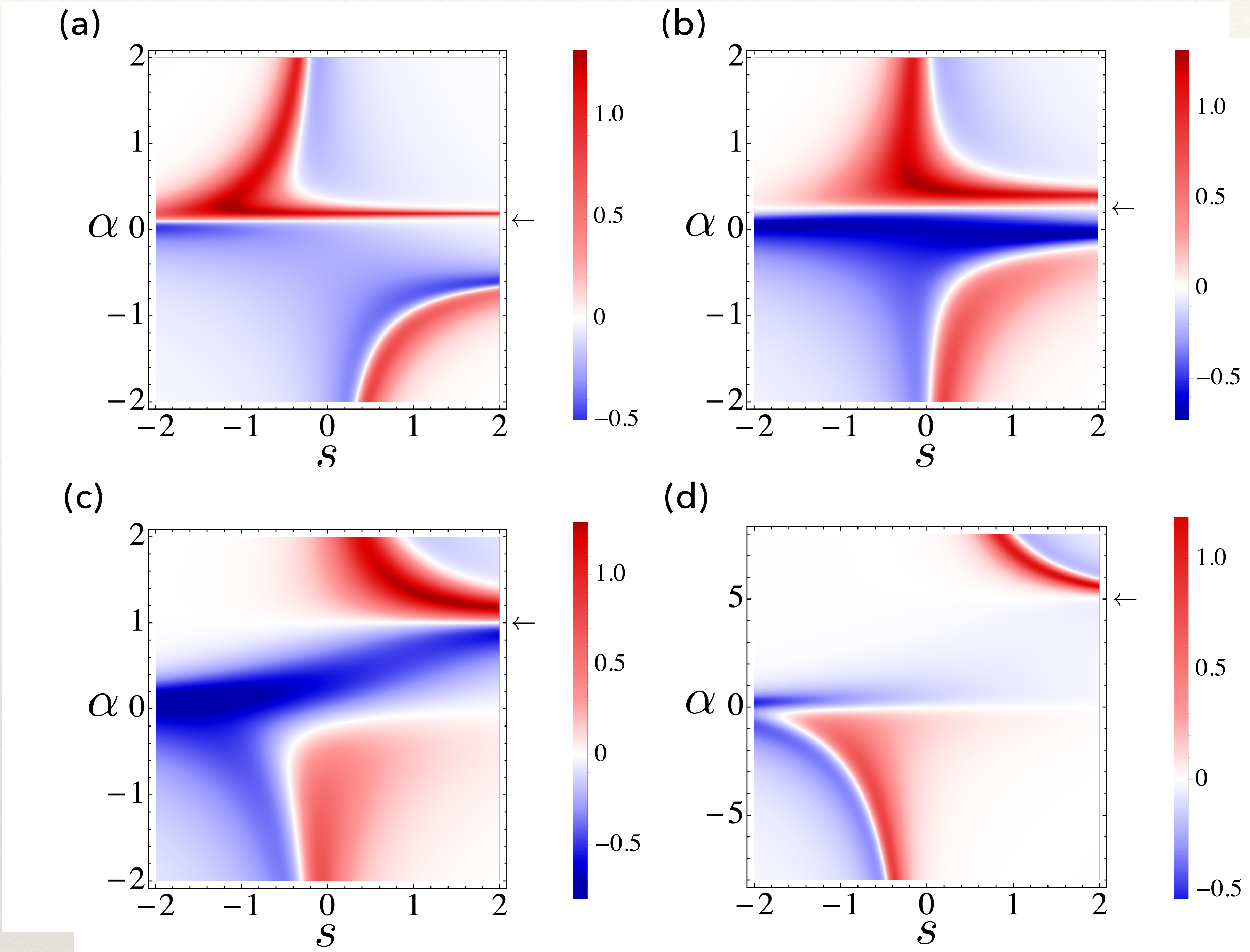


Figure 2. Mandel Q parameter $Q(s)$ as a function of α for $\Omega = 0.1$ (a), $\Omega = 0.25$ (b), $\Omega = 1$ (c) and $\Omega = 5$ (d) (we set $\gamma = 1$). Each curve was obtained through a mesh of the s-axis and numerical diagonalization of (13), from which $\theta(s)$ was inferred by selecting the largest eigenvalue. The s-dependent Mandel Q parameter $Q(s)$ was determined using Eqs. (16) after applying the finite difference method to compute the derivatives of $\theta(s)$.