Quantum tunneling:
Applications in Quantum Information

OUTLINE:

- *One- and two-particle: quantum state transfer & entanglement generation*
- *Many-body dynamics in quadratic models*
- *Applications: n-QST, quantum batteries, entanglement generation*

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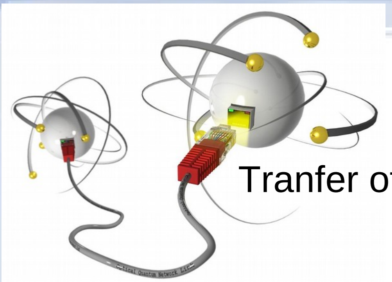
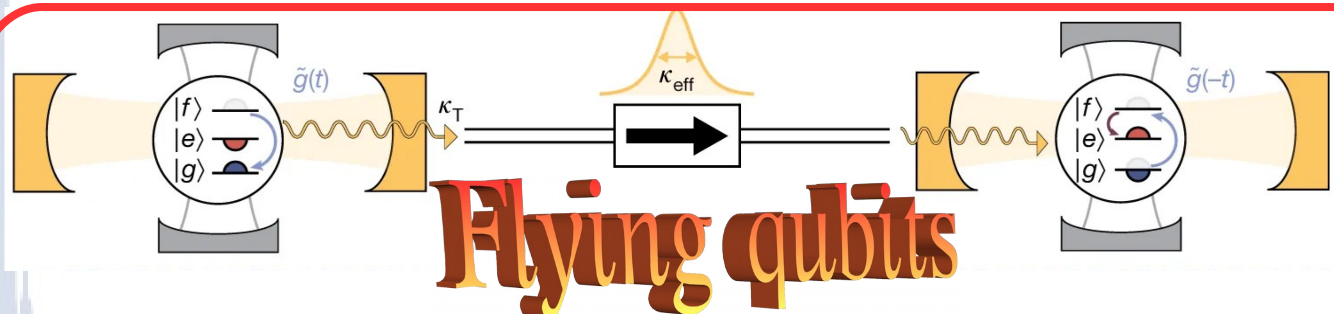
Quantum State Transfer (QST)



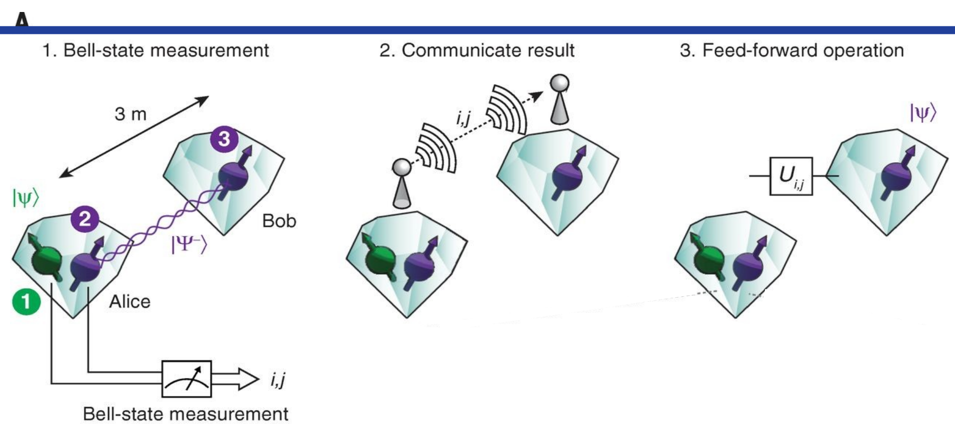
Tranfer of the “quantum information”, i.e., the quantum state, is mandatory in order to perform a QIP task.

The qubit: the elementary unit of quantum information $|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$



QST Fidelity: 80.02±0.07 %
Distance: 0.9 m
Protocol duration 180 ns
Entanglement Fidelity: 78.9% ±0.1
Concurrence: 0.747 ± 0.004
Rate: 50 kHz

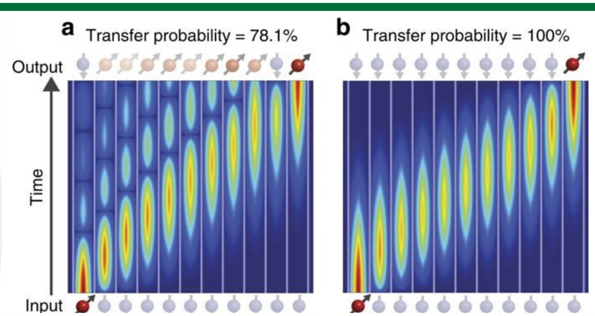
Kurpiers *et. al*, *Deterministic quantum state transfer and remote entanglement using microwave photons*, Nature **558**, 264-267 (2018)



QST Fidelity: 77±3 % Distance: 3 m

Pfaff *et al.*, *Unconditional quantum teleportation between distant solid-state quantum bits,* Science **345**, pp 532-535 (2014)

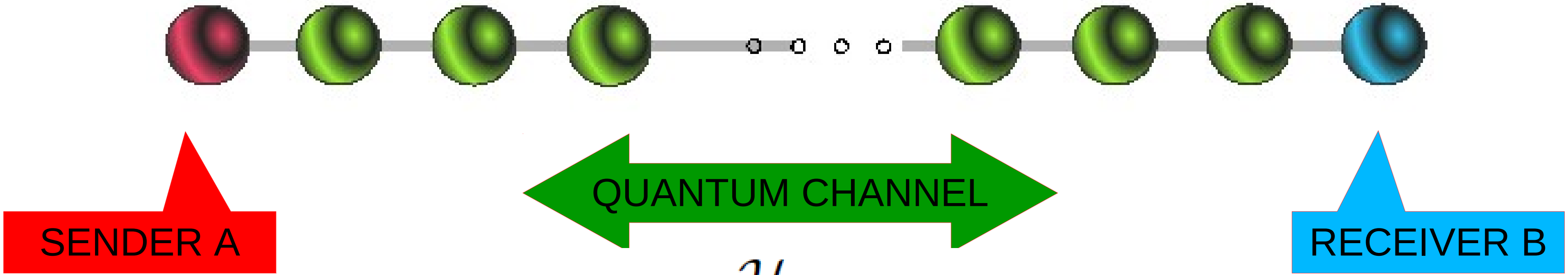
Teleportation



Chapman *et al*, *Experimental perfect state transfer of an entangled photonic qubit,* Nature Communications **7**, 11339 (2016)

Quantum Channel

QUANTUM-STATE TRANSFER (QST)



$$|\Psi\rangle_A |\ldots\rangle \xrightarrow{\mathcal{H}_{QC}} |\ldots\rangle |\Psi\rangle_B$$

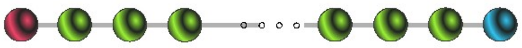
Hopping Hamiltonian

$$\hat{H}_c = \sum_{i=1}^{N} \frac{J_i}{2} \left(\hat{c}_{i+1}^{\dagger} \hat{c}_i + \hat{c}_i^{\dagger} c_{i+1} \right) + h_i \hat{c}_i^{\dagger} \hat{c}_i \ ,$$

Fermions - JW mapping - Spin-1/2 XX model $\hat{H} = \sum_{i}^{N} J_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y \right) + h_i \hat{S}_i^z$

Bosons - HP approx. - Large S Heisenberg model $\hat{H} = -J \sum_{\langle mn \rangle} \hat{\mathbf{S}}_m \cdot \hat{\mathbf{S}}_n$

QST IN THE XX MODEL



INPUT STATE OF THE QUBIT s → time → OUTPUT STATE OF THE QUBIT r

$$|\Psi\rangle_s = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \longrightarrow \rho_r(t) = \begin{pmatrix} 1-|\beta f_s^r(t)|^2 & \alpha\beta^* f_s^r(t)^* \\ \alpha^*\beta f_s^r(t) & |\beta f_s^r(t)|^2 \end{pmatrix}$$

The receiver's state depends only on the transition amplitude s → r

$$f_s^r(t) = \sum_{k=1}^{N} \langle r| e^{-it\hat{H}} |s\rangle = \sum_{k=1}^{N} e^{-i\omega_k t} a_{rk} a_{ks}^*$$

Quantifiers of the quality of the QST protocol

Fidelity

$$\mathcal{F}_{sr}(\theta,\phi;t) = \sqrt{{}_s\langle\Psi|\,\rho_r(t)\,|\Psi\rangle_s}$$

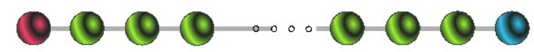
Average Fidelity

$$\bar{\mathcal{F}}_{sr}(t) = \frac{1}{\Omega}\int_\Omega d\Omega\, \mathcal{F}_{sr}(\theta,\phi;t)$$

The average fidelity depends only on (the modulus of) the transition amplitude

$$\bar{\mathcal{F}}_{sr}(t) = \frac{1}{2} + \frac{|f_s^r(t)|}{3} + \frac{|f_s^r(t)|^2}{6}$$

LONG SPIN CHAINS



Hamiltonian engineering

$$J_i = \frac{\pi}{N+1}\sqrt{i(N-i)}$$

Linear spectrum
No dispersion

$$\omega_k = ck$$

P-QST

$$\bar{\mathcal{F}}_{sr}(t^*) = 1$$

$$t^* = N$$

Christandl *et al.*, Phys. Rev. Lett. **92**, 187902 (2004); Di Franco *et al.*, Phys. Rev. Lett. **101**, 230502 (2008); Pitsios *et al.*, Nature Communications **8**, 1569 (2017)

Perturbative couplings

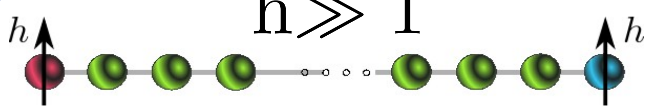
j≪ 1

$$\bar{\mathcal{F}}_{sr}(t^*) = 1 - O\left(Nj^2\right)$$

$$t_e^* = j^{-2} \quad \text{even } N$$

$$t_o^* = \frac{\sqrt{N}}{j} \quad \text{odd } N$$

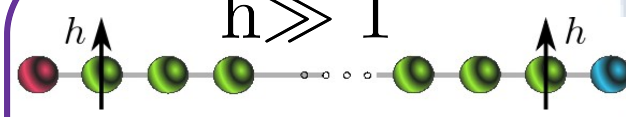
h≫ 1



$$\bar{\mathcal{F}}_{sr}(t^*) = 1 - O\left(h^{-2}\right)$$

$$t^* = h^N$$

h≫ 1



$$\bar{\mathcal{F}}_{sr}(t^*) = 1 - O\left(Nh^{-2}\right)$$

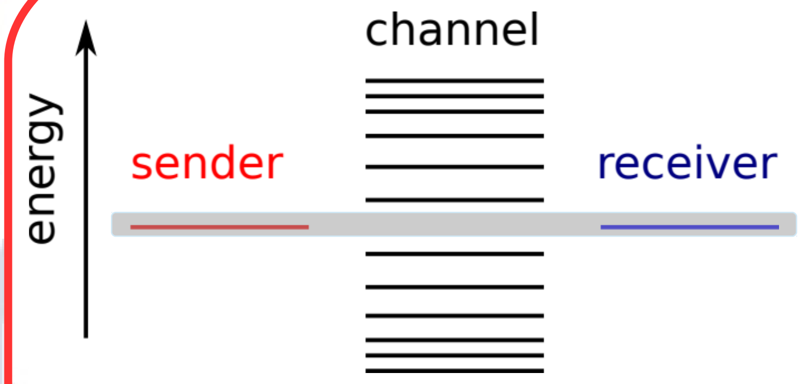
$$t_e^* = h^2 \quad \text{even } N$$

$$t_o^* = \sqrt{N}h^{-1} \quad \text{odd } N$$

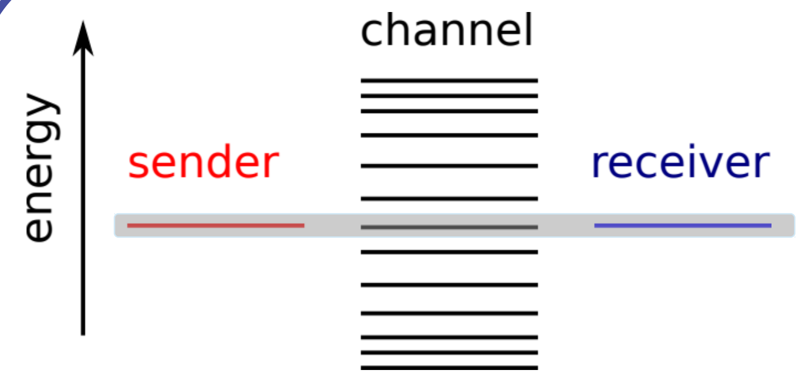
Perturbtive couplings reduce the effective Hilbert space to a 2 (or 3) level system.

Wójcik et al., Phys. Rev. A **72**, 034303 (2005), Plastina and Apollaro, Phys. Rev. Lett. **99**, 177210 (2007)

on-resonant vs off-resonant tunneling



$$E = 0 \ , \quad \{|1\rangle_s ; |1\rangle_r\}$$



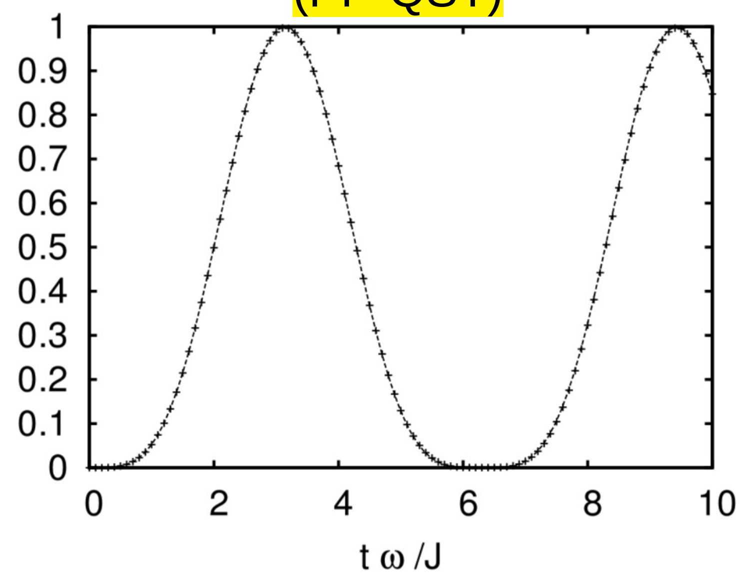
$$E = 0 \ , \quad \{|1\rangle_s ; |\Psi\rangle_c ; |1\rangle_r\}$$

effective 2-level system

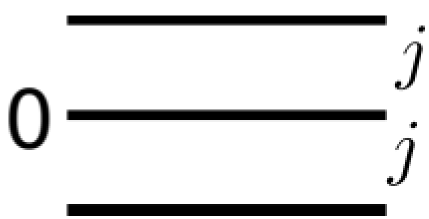
$$|\Psi_1\rangle \simeq \frac{1}{\sqrt{2}}(|1\rangle + |N\rangle)$$

$$|\Psi_2\rangle \simeq \frac{1}{\sqrt{2}}(|1\rangle - |N\rangle)$$

perturbatively-perfect QST (PP-QST)



effective 3-level system



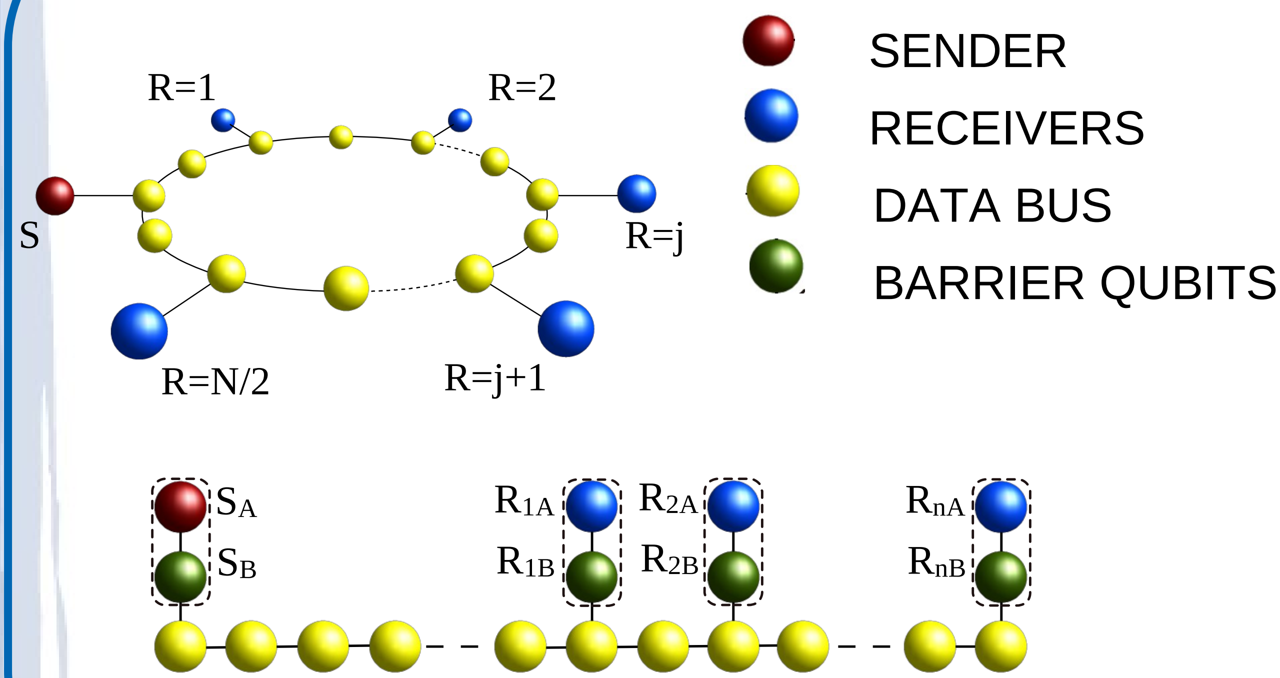
Applications of PP-QST

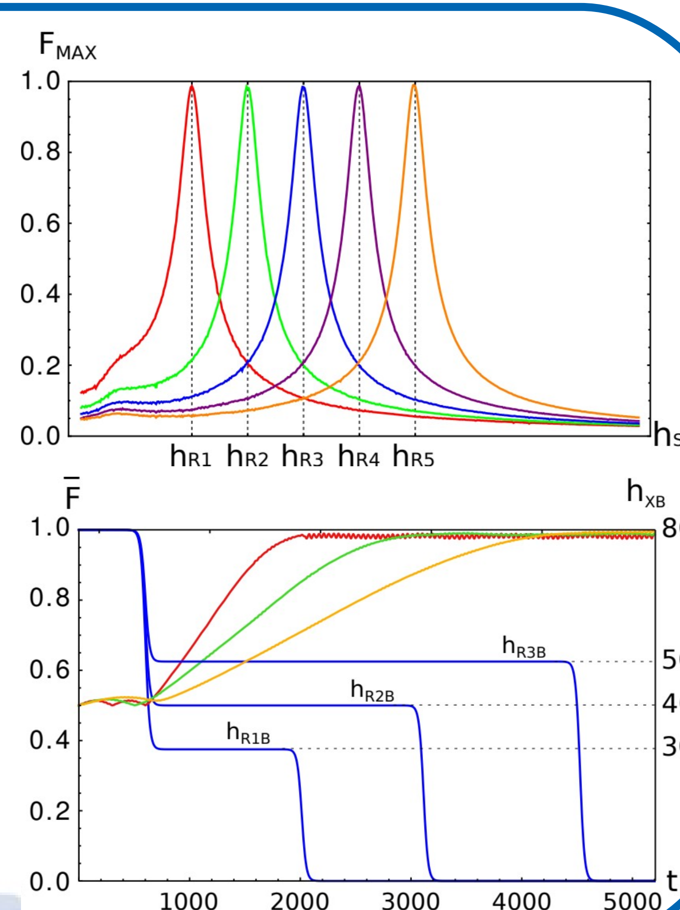
Perturbative entangling gate

$$|10\rangle \xrightarrow{t=t^*/2} \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) \xrightarrow{t=t^*/2} |01\rangle$$

Banchi et al., Phys. Rev. Lett. **106**, 140501 (2010)

Quantum State Routing





Paganelli et al., Phys. Rev. A **87**, 062309 (2013)

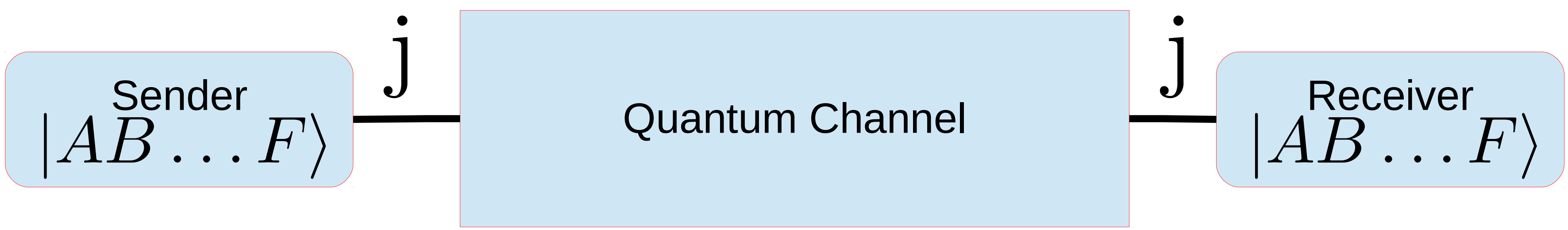
A single channel for multiple QIP tasks

Motivations:

- The technological challenge of faithful quantum wire;
- The request of scalability of a quantum computer;
- The short coherence times of the coherent dynamics;
- The protection against environmental intrusions;
- The economical costs of a single quantum wire;
- ...

Can perturbative couplings be helpful in this regard?

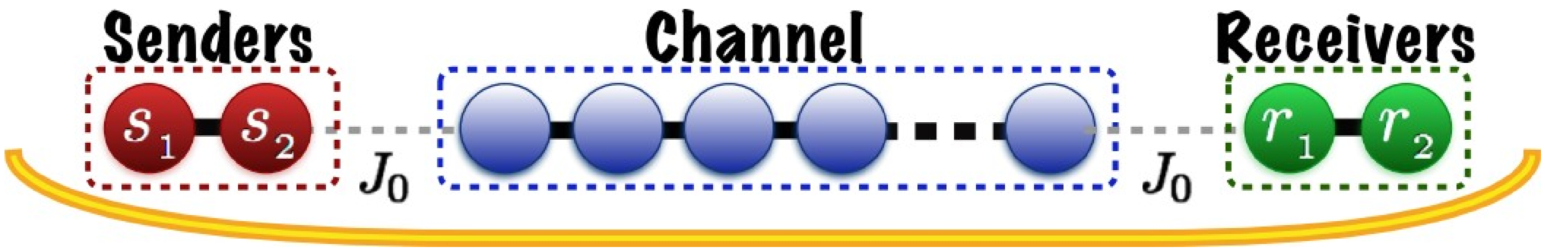
Task: Many-body quantum state transfer



Motivations:
the output of a QIP protocol is a n-qubit state
transfer of multipartite entanglement
many-body properties transfer

Alternative Protocols:
parallel/sequential use of a 1-QST
use of entangled states as QC
PQST QC

n-QST in spin chains with U(1) symmetry



$$|\Psi(0)\rangle_{12..n} = a_0\,|0\rangle + \sum_{n_1=1}^{n} a_{n_1}\,|n_1\rangle + \sum_{n_1<n_2=1}^{n} a_{n_1 n_2}\,|n_1 n_2\rangle + \ldots$$

senders + receivers + quantum channel initial state

$$\ldots + \sum_{n_1<n_2<\ldots<n_j=1}^{n} a_{n_1 n_2 \ldots n_j}\,|n_1 n_2 \ldots n_j\rangle + \ldots + a_{n_1 n_2 \ldots n_n}\,|n_1 n_2 \ldots n_n\rangle$$

initial senders state

Hamiltonian

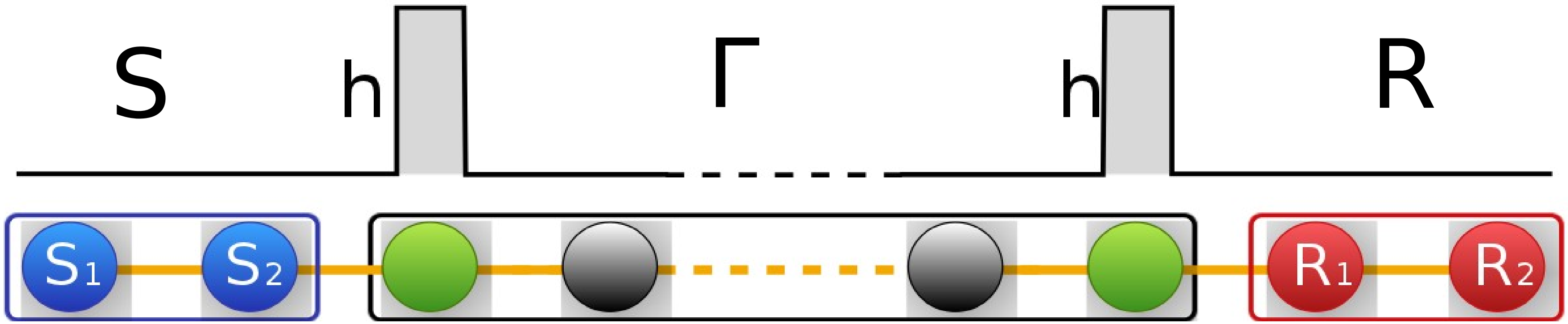
$$\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)} + \hat{H}^{(2)} + \cdots + \hat{H}^{(N)}$$

$$|\Psi(t)\rangle_{12..N} = a_0\,|0\rangle + \sum_{n_1,m_1,k_1=1}^{N} a_{n_1}(t) D_{n_1}^{k_1} U_{k_1}^{m_1}\,|m_1\rangle$$

senders + receivers + channel evolved state

$$+ \sum_{n_1<n_2;k_1<k_2;n,m=1}^{N} a_{n_1 n_2}(t) D_{n_1 n_2}^{k_1 k_2} U_{k_1}^{m_1} U_{k_2}^{m_2}\,|m_1 m_2\rangle + \ldots + \sum_{n^\uparrow=1}^{N} \sum_{k^\uparrow=1}^{N} \sum_{m_i=1}^{N} a_{n^\uparrow} D_{k_i^\uparrow}^{n_i^\uparrow} U_{m_i}^{k_i^\uparrow}\,\left|\{m_i^\uparrow\}\right\rangle$$

2-QST IN U(1) SYMMETRY CONSERVED MODELS



sender state $$|\Psi(0)\rangle_{12} = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle$$

receivers density matrix

$$\begin{pmatrix} \mathcal{G}_N^{N-1}\mathcal{G}^{*}{}_N^{N-1} & \mathcal{F}^{*}{}_{N-1}\mathcal{G}_N^{N-1} & \mathcal{F}^{*}{}_{N}\mathcal{G}_N^{N-1} & \alpha^{*}\mathcal{G}_N^{N-1} \\ \mathcal{F}_{N-1}\mathcal{G}^{*}{}_N^{N-1} & |\mathcal{G}_{N-1}^{m}|^2+|\mathcal{F}_{N-1}|^2 & \mathcal{G}_{N-1}^{m}\mathcal{G}^{*}{}_N^{m} + \mathcal{F}_{N-1}\mathcal{F}^{*}{}_N & \mathcal{F}^{*}{}_m\mathcal{G}_{N-1}^{m}+\alpha^{*}\mathcal{F}_{N-1} \\ \mathcal{F}_{N}\mathcal{G}^{*}{}_N^{N-1} & \mathcal{G}_N^{m}\mathcal{G}^{*}{}_{N-1}^{m} + \mathcal{F}_N\mathcal{F}^{*}{}_{N-1} & |\mathcal{G}_N^{m}|^2+|\mathcal{F}_N|^2 & \mathcal{F}^{*}{}_N\mathcal{G}_N^{m}+\alpha^{*}\mathcal{F}_N \\ \alpha\mathcal{G}^{*}{}_N^{N-1} & \mathcal{F}_m\mathcal{G}^{*}{}_{N-1}^{m}+\alpha\mathcal{F}^{*}{}_{N-1} & \mathcal{F}_m\mathcal{G}^{*}{}_N^{m}+\alpha\mathcal{F}^{*}{}_N & 1-|\mathcal{G}_{N-1}^{m}|^2-|\mathcal{G}_N^{m}|^2-|\mathcal{F}_{N-1}|^2-|\mathcal{F}_N|^2 \end{pmatrix}$$

1-excitation transition amplitude

$$\mathcal{F}_r = \beta\,\langle r|\,U\,|1\rangle + \gamma\,\langle r|\,U\,|2\rangle$$

2-excitation transition amplitude

$$\mathcal{G}_s^r = \delta\,\langle s, r|\,U\,|1, 2\rangle$$

2-QST IN U(1) SYMMETRY CONSERVED MODELS

$$\bar{F}(t) = \frac{1}{4} + \frac{5}{54} Re\left[f_{s_1}^{r_1} + f_{s_2}^{r_2} + \frac{7}{5} f_{s_2}^{r_2}(f_{s_1}^{r_1})^*\right] + \frac{1}{54}\left(|f_{s_2}^{r_1}|^2 + |f_{s_1}^{r_2}|^2\right) + \frac{5}{108}\left(|f_{s_2}^{r_2}|^2 + |f_{s_1}^{r_1}|^2\right)$$

$$+ \frac{7}{54} Re\left[g_{s_1 s_2}^{r_1 r_2}\right] + \frac{5}{108}|g_{s_1 s_2}^{r_1 r_2}|^2 - \frac{1}{54}\left(1 - \sum_{n<m=1}^{n,m \neq \mathcal{R}} |g_{s_1 s_2}^{nm}|^2\right)$$

$$+ \frac{5}{54} Re\left[\left(f_{s_1}^{r_1} + f_{s_2}^{r_2}\right)\left(g_{s_1 s_2}^{r_1 r_2}\right)^*\right] - \frac{1}{27}\sum_{n=1}^{n \neq \mathcal{R}} Re\left[(f_{s_2}^{n})^* g_{s_1 s_2}^{n r_1} + (f_{s_1}^{n})^* g_{s_1 s_2}^{n r_2}\right]$$

CONSTANT TERM

SINGLE PARTICLE TRANSFER AMPLITUDE

TWO-PARTICLE TRANSFER AMPLITUDE

INTERFERENCE BETWEEN SINGLE- AND TWO-PARTICLE TRANSFER AMPLITUDES

$$\mathcal{F}_{LOCC} = \frac{2}{d+1} \longrightarrow \mathcal{F}_{LOCC}^{2qb} = \frac{2}{5} = 0.4$$

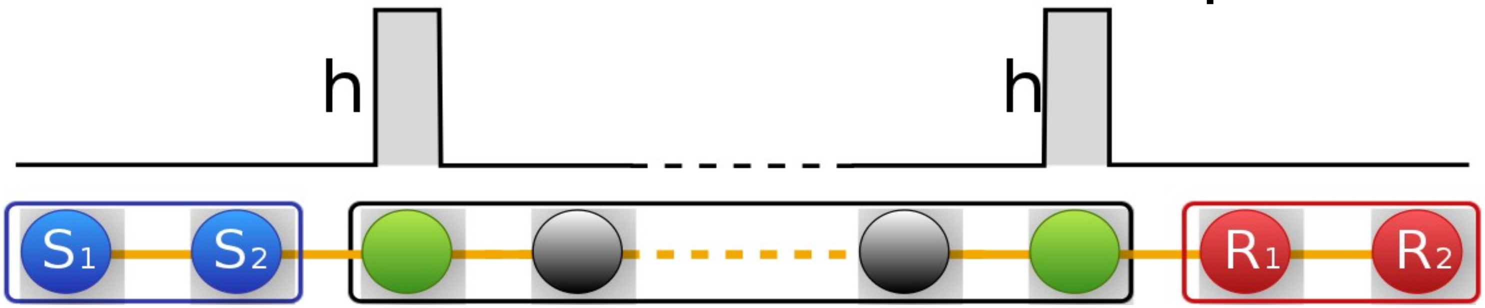
$$\mathcal{F}_{UQCM} = \frac{d+2}{2(d+1)} \longrightarrow \mathcal{F}_{UQCM}^{2qb} \frac{3}{5} = 0.6$$

2-QST IN U(1) & BILINEAR MODELS

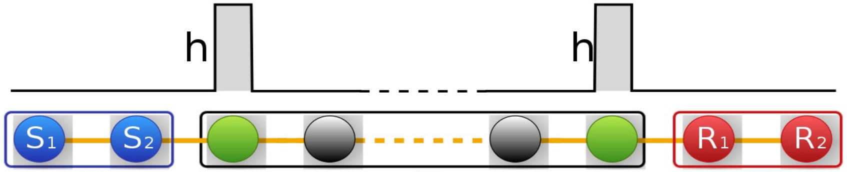
XX SPIN-1/2 MODEL $$H=\sum_{l=1}^{N-1}\left(\sigma_l^x\sigma_{l+1}^x+\sigma_l^y\sigma_{l+1}^y\right)+h\left(\sigma_3^z+\sigma_{N-2}^z\right)$$

BILINEAR SPINLESS FERMION MODEL $$H=\sum_{i=1}^{N-1}c_i^\dagger c_{i+1}+c_{i+1}^\dagger c_i+h\left(c_3^\dagger c_3+c_{N-2}^\dagger c_{N-2}\right)$$

The 2-excitation transition amplitude can be expressed as a determinant of 1-excitation transition amplitudes

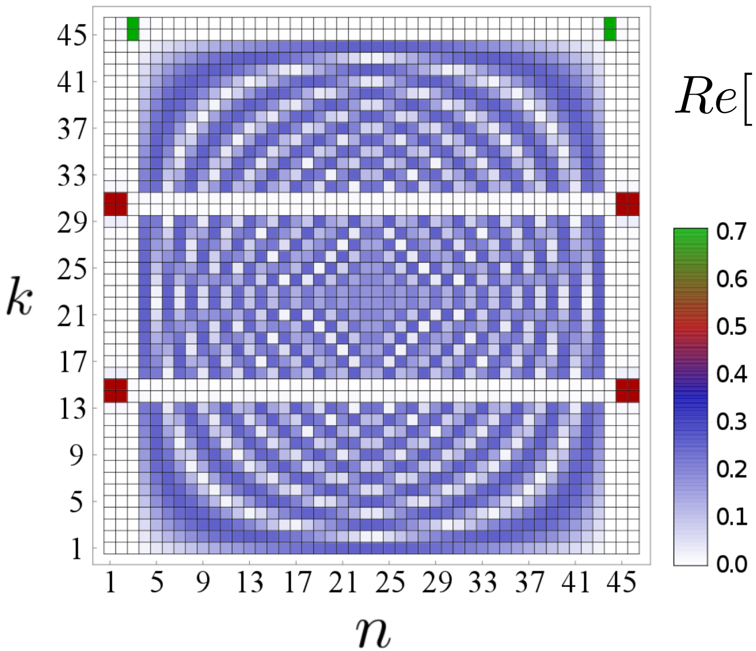


$$\bar{F}_a(t)=\frac{1}{4}+\frac{10}{54}Re\left[f_1^{N-1}\right]+\frac{7}{54}Re\left[\left(f_1^{N-1}\right)^2\right]+\frac{12}{54}|f_1^{N-1}|^2+\frac{2}{54}|f_1^N|^2$$
$$+\frac{10}{54}|f_1^{N-1}|^2Re\left[f_1^{N-1}\right]-\frac{10}{54}Re\left[f_1^{N-1^*}f_1^Nf_2^{N-1}\right]-\frac{7}{54}Re\left[f_1^Nf_2^{N-1}\right]$$

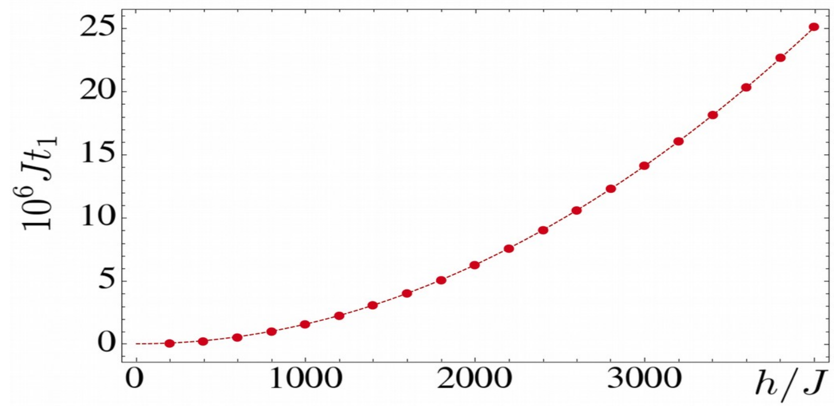


The average fidelity depends only on *one* single-transition amplitude

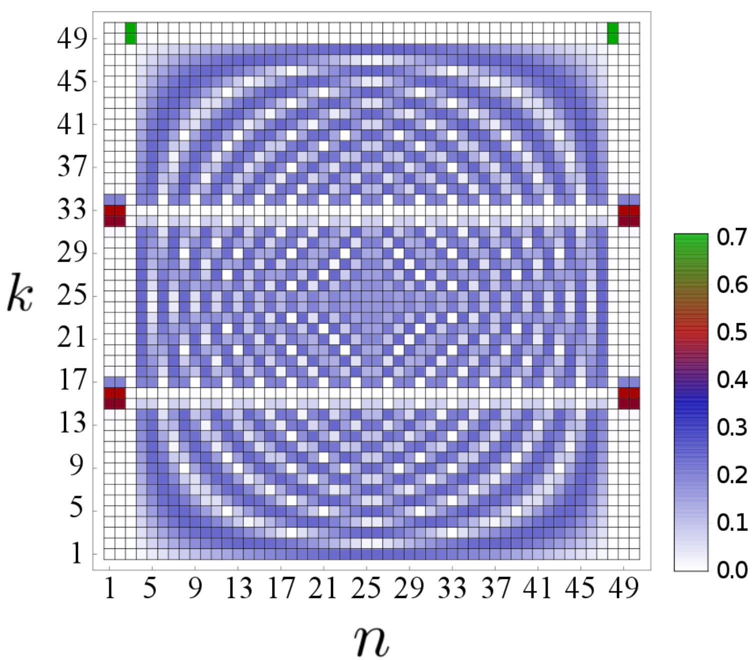
$$Re[f_1^{N-1}] = Re[\sum_{k=1}^{N} e^{-i\varepsilon_k t} a_{k1} a_{kN-1}] \simeq Re[\sum_{i=1}^{4} e^{-i\varepsilon_{q_i} t} a_{q_i 1} a_{q_i N-1}]$$



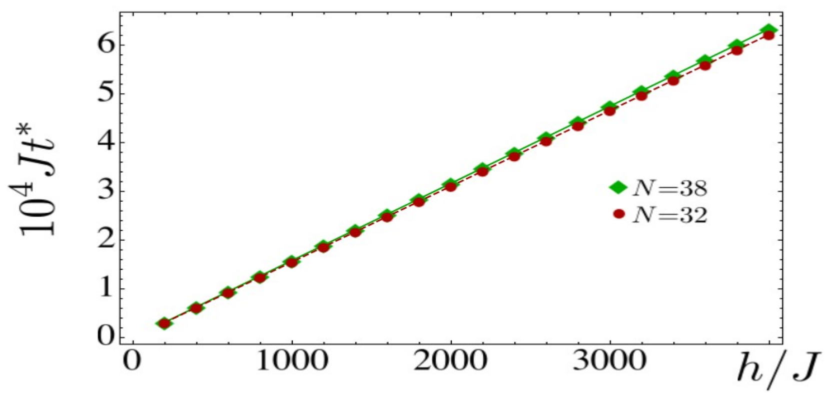
N ≠ 3n+2



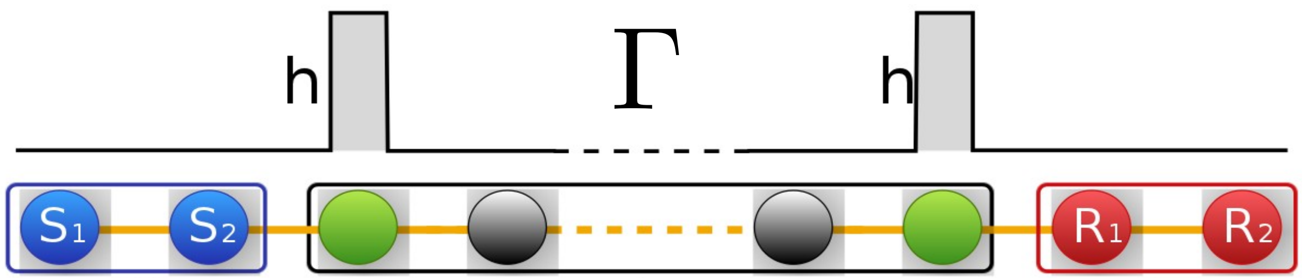
$$Re[f_1^{N-1}] \simeq Re[\sum_{i=1}^{6} e^{-i\varepsilon_{q_i} t} a_{q_i 1} a_{q_i N-1}]$$



N = 3n+2



2-QST IN THE XX SPIN-1/2 MODEL

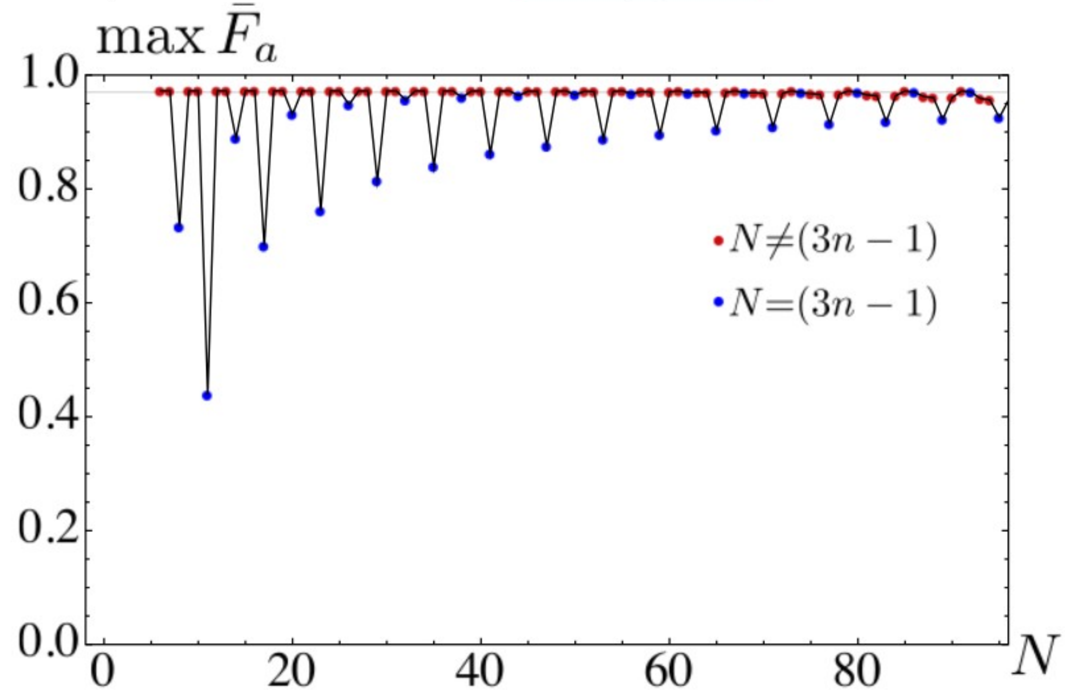


FINITE SIZE EFFECTS
DISAPPEAR FOR N>>1

$$N = 3n + 2 \longrightarrow N \neq 3n + 1$$

$$\langle \Psi_{s1s2}(0) | \, \Gamma \, \rangle \simeq O(N^{-1})$$

BASIC MECHANISM
IN A NUTSHELL



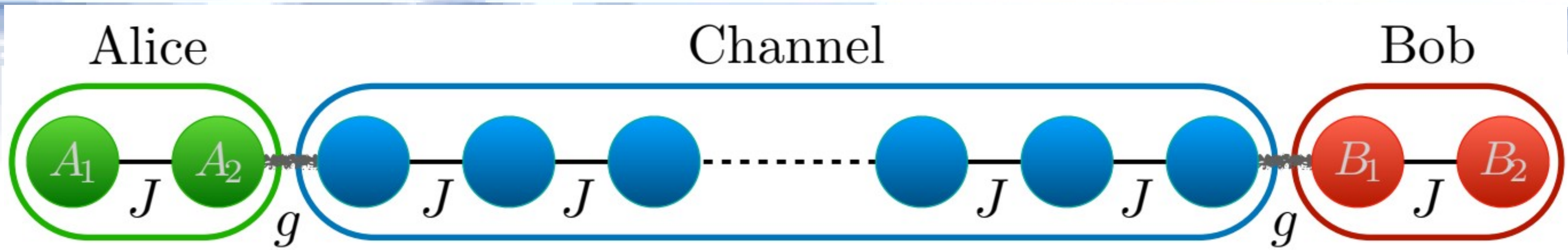
0-th order Hamiltonian with degenerate eigenstates

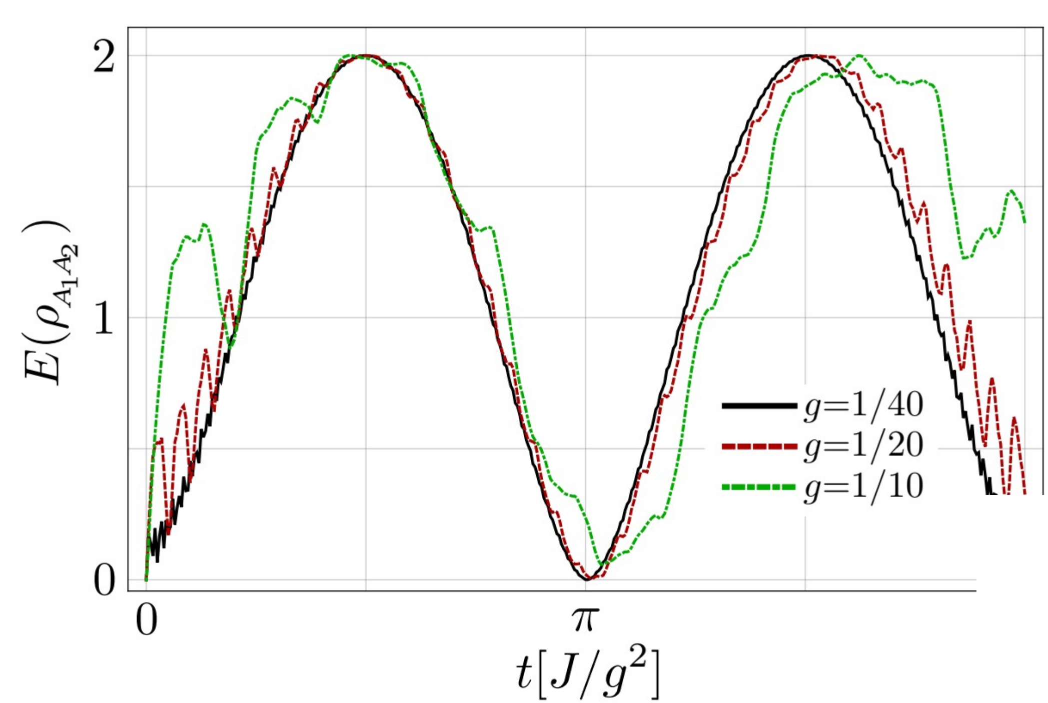
$$|00\rangle_{12} \quad |11\rangle_{12}$$

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_R$$

$$\mathcal{H}_S = \mathcal{H}_R = \sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y$$

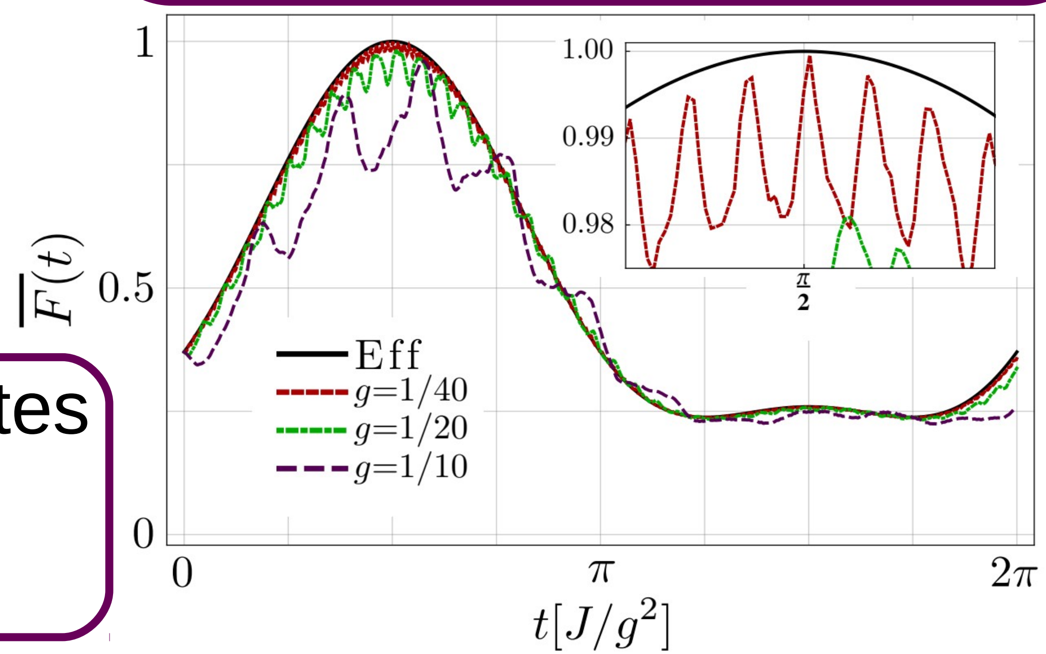
$$|\Psi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle)$$





Initial states evolving into Bell states

$$\{|11\rangle_s\,|00\rangle_r\,;|10\rangle_s\,|10\rangle_r\}\rightarrow$$
$$\left|\Phi^1_{\theta_n}\right\rangle_{A_1B_2(1)}\otimes\left|\Phi^2_{\theta_n}\right\rangle_{A_2B_1(2)}$$



Tensor product of n Bell states is a resource for n-qubit teleportation

n-excitation transfer in U(1) & bilinear models

$$\mathcal{F}=\begin{pmatrix} f_1^1 & f_1^2(t) & \cdots & f_1^{N-3} & f_1^{N-1} & f_1^N \\ f_2^1 & \cdots & \cdots & f_2^{N-3} & f_2^{N-2} & f_2^N \\ f_3^1 & \cdots & \cdots & f_3^{N-3} & f_3^{N-2} & f_3^N \\ \vdots & & \ddots & & & \vdots \\ f_N^1 & & & & \cdots & f_N^N \end{pmatrix}$$

n=1

n=2

n=3

n-excitation transfer amplitude is given by the determinant (permanent) of the minor for fermions (bosons).

n-particle dynamics of fermions and bosons show identical behaviour w.r.t. transfer time and transition amplitude

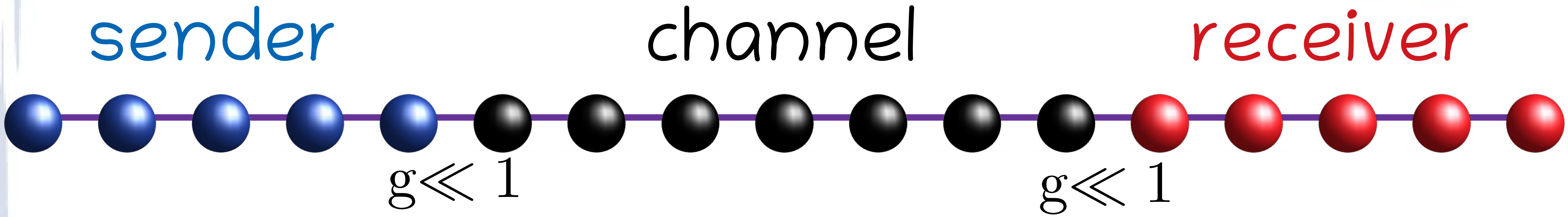
n-excitation transfer in U(1) & bilinear models

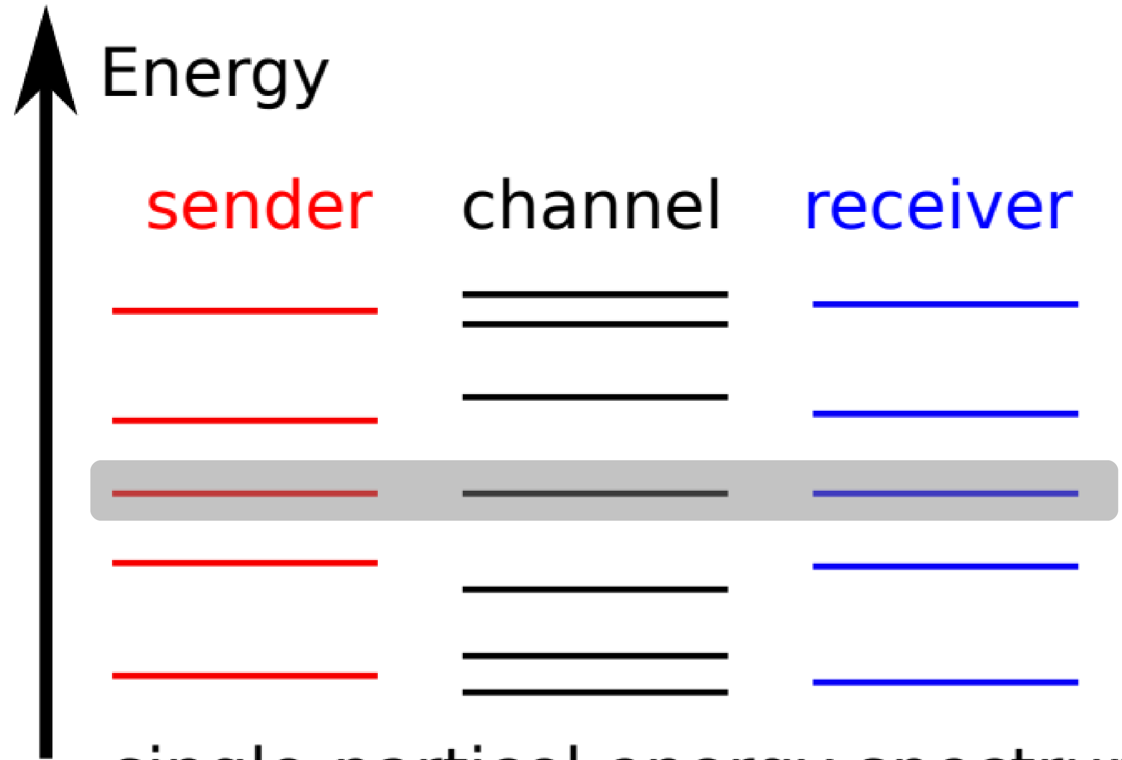
$$\mathcal{F}(t) = \begin{pmatrix} f_1^{N-4} & f_1^{N-3} & f_1^{N-1} & f_1^{N} \\ f_2^{N-4}s & f_2^{N-3} & f_2^{N-2} & f_1^{N-1} \\ f_3^{N-4} & f_3^{N-3} & f_2^{N-3} & f_1^{N-3} \\ f_4^{N-4} & f_3^{N-4} & f_2^{N-4} & f_1^{N-4} \end{pmatrix} ,$$

For n-excitation transfer we need to maximise Ceiling[n/2] 1-particle transition amplitudes at t*

$$f_s^r(t) = \sum_{k=1}^{N} \langle r| e^{-it\hat{H}} |s\rangle = \sum_{k=1}^{N} e^{-i\omega_k t} a_{rk} a_{ks}^*$$

where N is, perturbatively, the number of **resonant** modes





single-particel energy spectrum

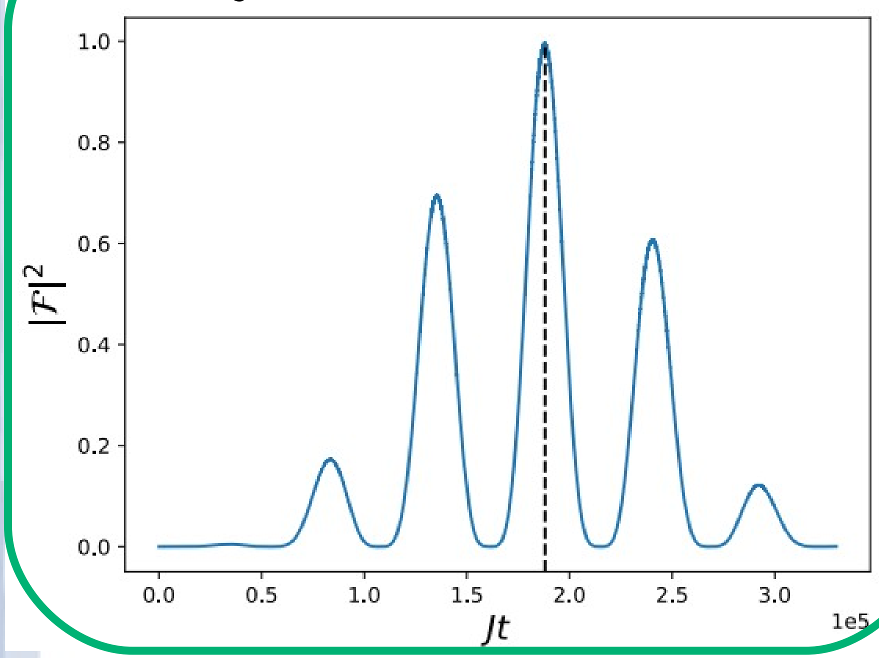
Resonance condition

$$\frac{k\pi}{n_s+1} = \frac{q\pi}{n_w+1} \;, \; k = 1, .., n_s \text{ and } q = 1, .., n_w \;.$$

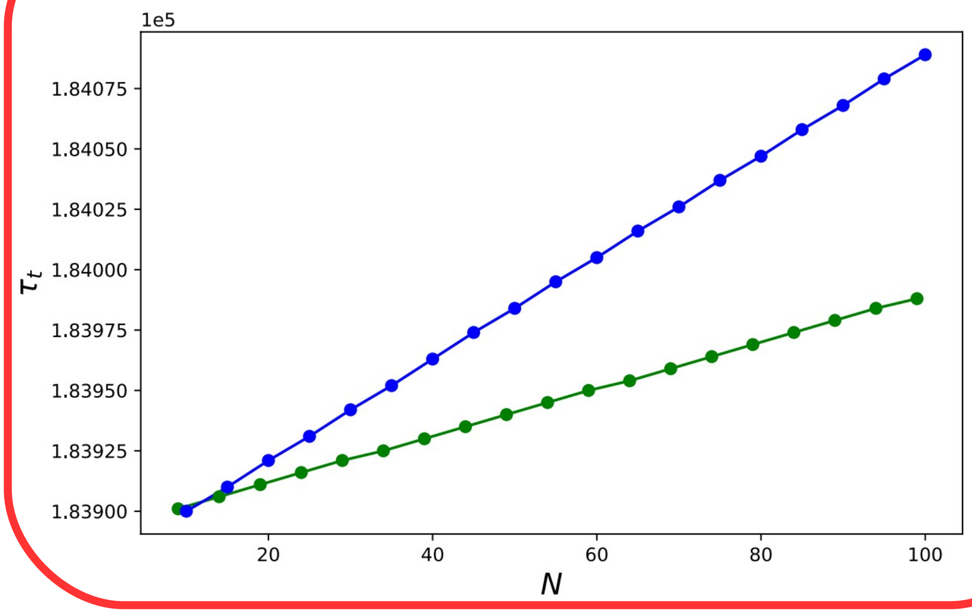
Length of wires that are equivalent mod(number of senders) have the same behaviour w.r.t. excitation transfer

Number of Excitations	Number of Resonant Modes	Length of the wire
1	0 1	$2n$ $2n+1$
2	0 0 2	$3n$ $3n+1$ $3n+2$
3	0 1 0 3	$4n$ $4n+1$ $4n+2$ $4n+3$
4	0 0 0 0 4	$5n$ $5n+1$ $5n+2$ $5n+3$ $5n+4$
5	0 1 2 1 0 5	$6n$ $6n+1$ $6n+2$ $6n+3$ $6n+4$ $6n+5$

Fidelity of 4-excitation transfer



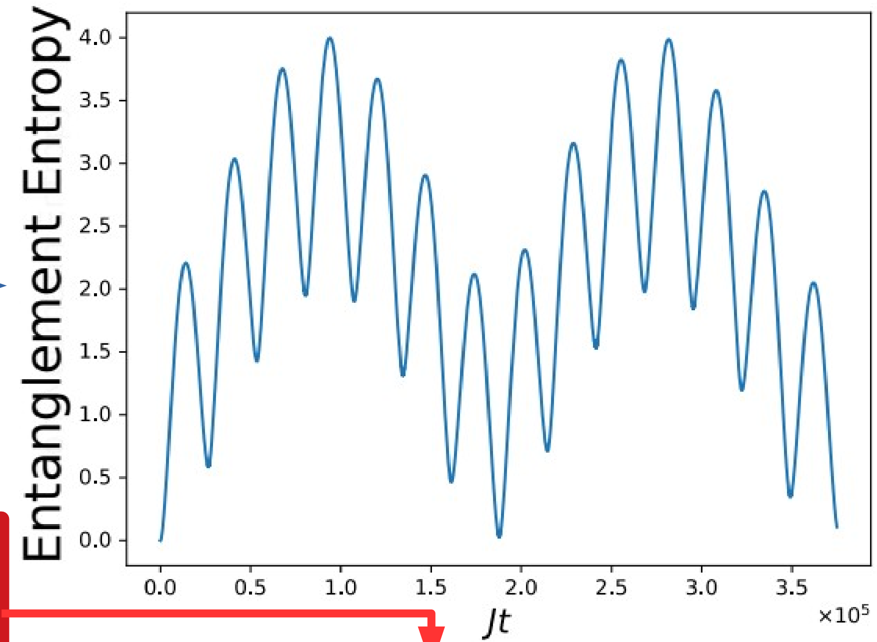
Linear increase of the transfer time with N



n-particle dynamics useful for:

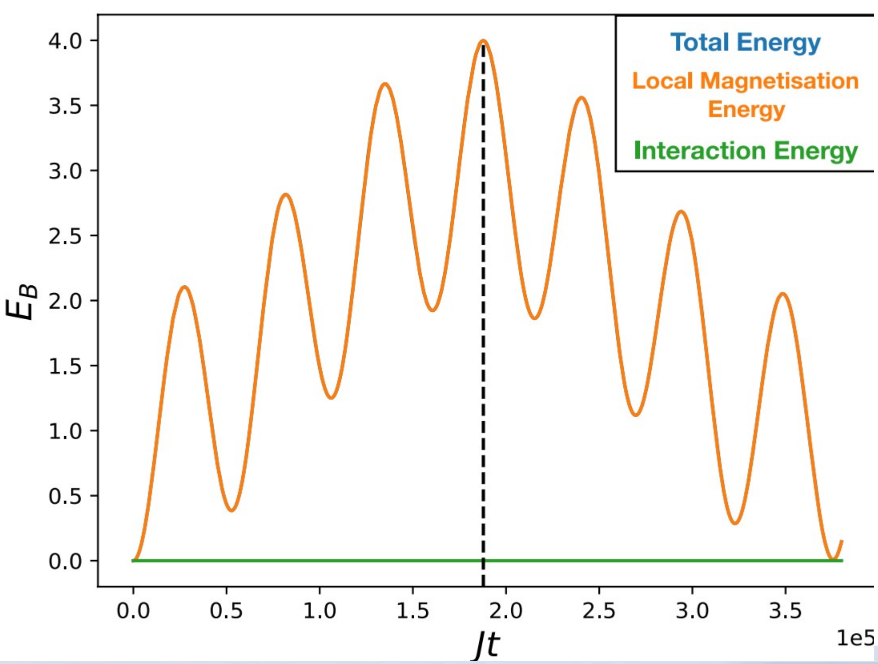
sender channel receiver

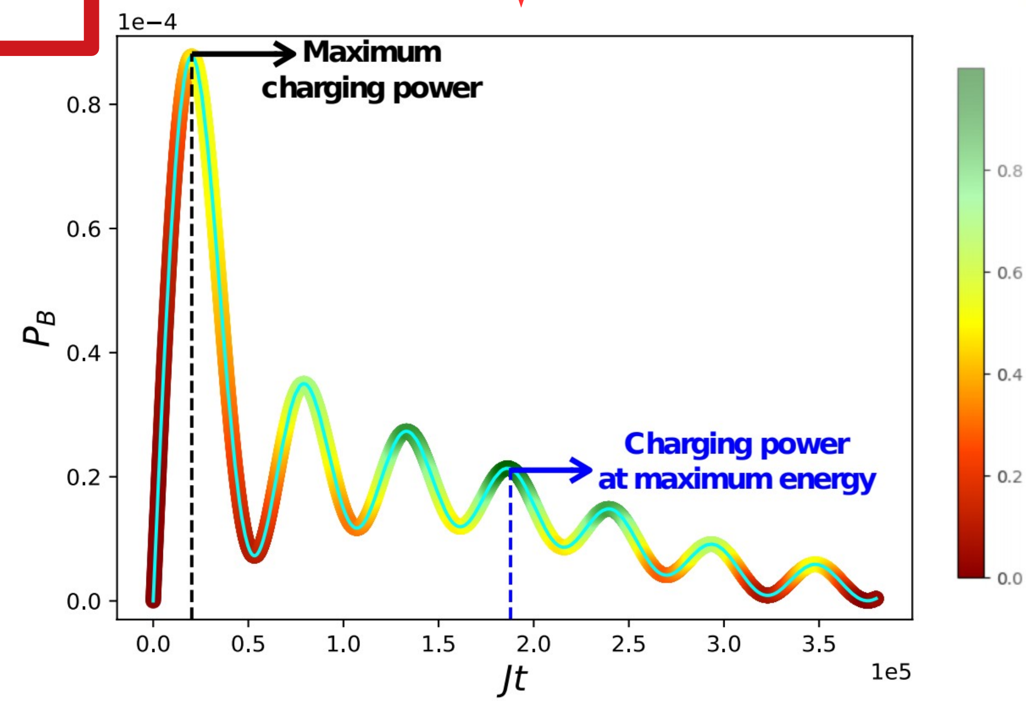
Multi-qubit bipartite entanglement generation



Energy transfer

Quantum battery charging





CONCLUSIONS

- n-QST protocol over *universal* quantum spin chain
- Perturbatively-perfect n-QST
- Applications to quantum batteries and multi-qubit bipartite entanglement generation

<u>Outlooks:</u>

- Faster (ballistic?) n-QST
- N-QST in U(1) interacting Hamiltonians
- Multipartite entanglement

Lorenzo, Apollaro, Paganelli, Palma, Plastina, Phys. Rev. A **91**, 042321 (2015)
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Apollaro, Almeida, Lorenzo, Ferraro, Paganelli, arXiv:1812.11609

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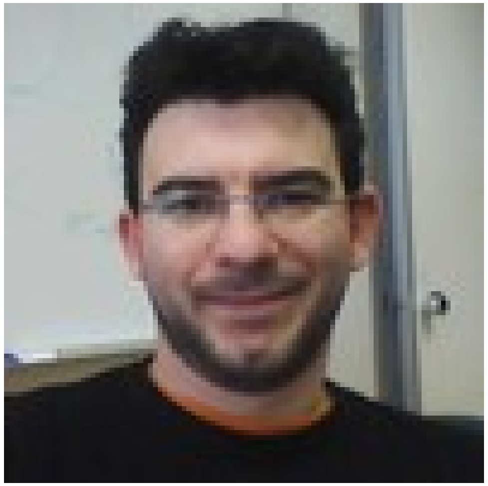

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CONCLUSIONS

- n-QST protocol over quantum spin chain
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THANK YOU FOR YOUR ATTENTION

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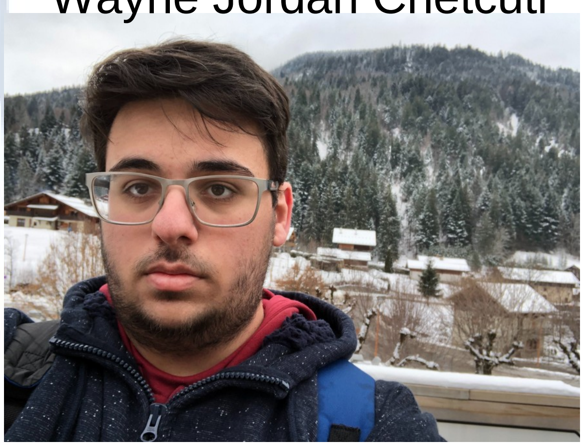

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